Operational Amplifiers, Version 1.5

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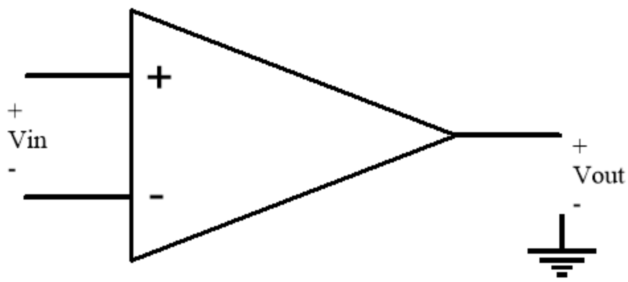
Scope

I initially wrote this article to help community college students solve three homework problems related to opamps. They had only recently learned Ohm's law and nodal equations and had not been introduced to the concept of feedback. Although feedback is central to understanding Operational Amplifiers, I do not address it head-on. Instead, I built on what they already knew.

Contents

The Ideal Operational Amplifier



An ideal Operational Amplifier, or opamp, is a deceptively simple device. You apply V_{in}, and it drives V_{out}.

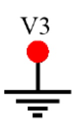
$$V_{out} = k \times V_{in} \quad (1)$$

where *k* is just a number with no units.

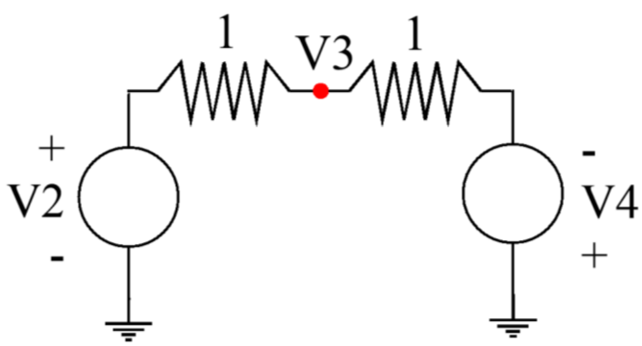
Equation 1 both tells you everything and nothing at the same time.

Starting From the Basics

Let me step back a bit and start with the basics.



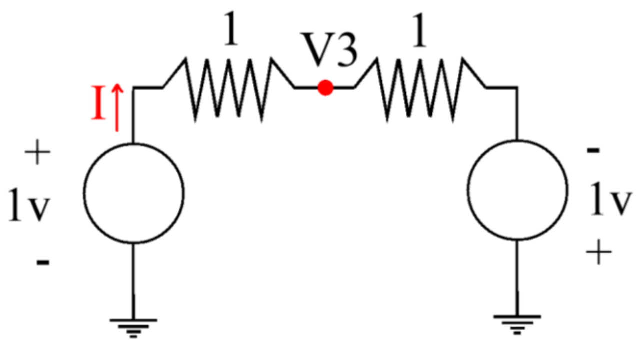
I have node 3 with a voltage, V_3, which I have grounded. The voltage on V_3 relative to ground is zero. It doesn't get more basic than this.



Rather than ground this node, I can attach two voltages sources, V_2 and V_4. They are connected to node 3 through 1 ohm resistors. If V_2 and V_4 are both zero, V_3 is at zero volts[2].

I know, it is still obvious.

[2] From now on, assume the voltage is relative to ground until otherwise noted.

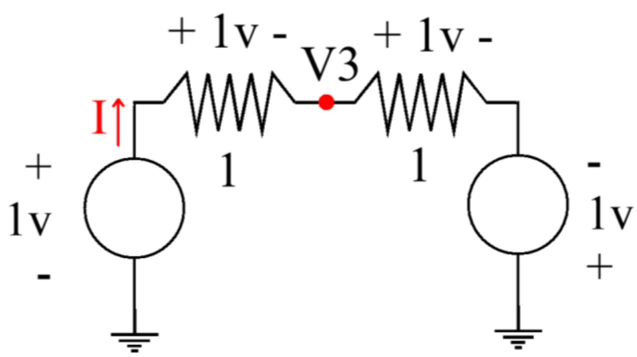


What if both sources are 1 volt? The current, I, can be calculated by finding the total voltage in the circuit and the total resistance.

$$I = \frac{1v+1v}{1\Omega+1\Omega} \qquad (2)$$

$$I = \frac{2\,v}{2\Omega} \qquad (3)$$

$$I = 1\ ampere$$

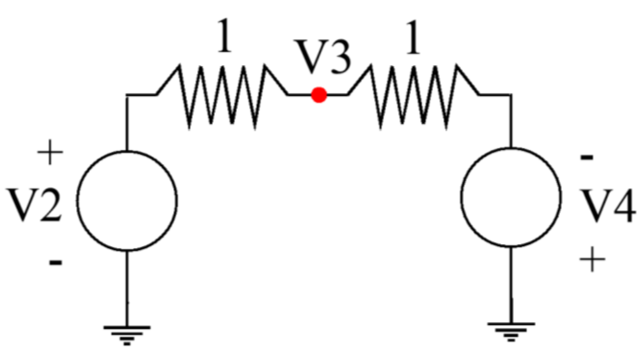


Knowing the current, I can calculate the voltage drop across each resistor

$$V_R = (1\ ampere) \times 1\ \Omega \qquad (4)$$

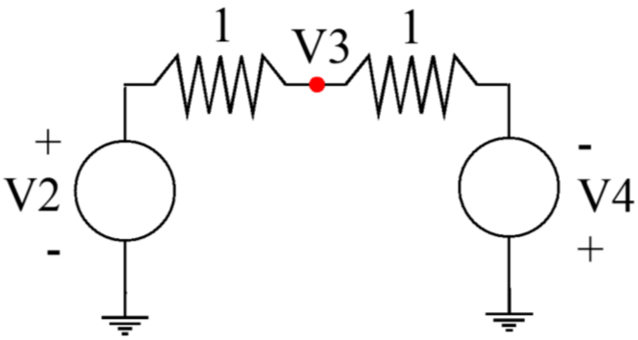
$$V_R = 1v$$

Starting at ground at the lower-left corner of the schematic, I have a voltage rise of 1 volt due to the left voltage source. Then I have a 1 volt voltage drop across the left resistor. This puts me at node 3 with a voltage of +1v – 1v = 0. So V_3 is at the same voltage as when I connected ground to it. I can call this situation "virtual ground" because measuring V_3 could be ground or just zero volts. Your voltmeter can't tell them apart.



Going back to the general case, as long as V_4 equals V_2, V_3 will always be at 0 volts.

Introducing Feedback



Now is when the magic begins. Say I define a relationship between V_4 and V_3.

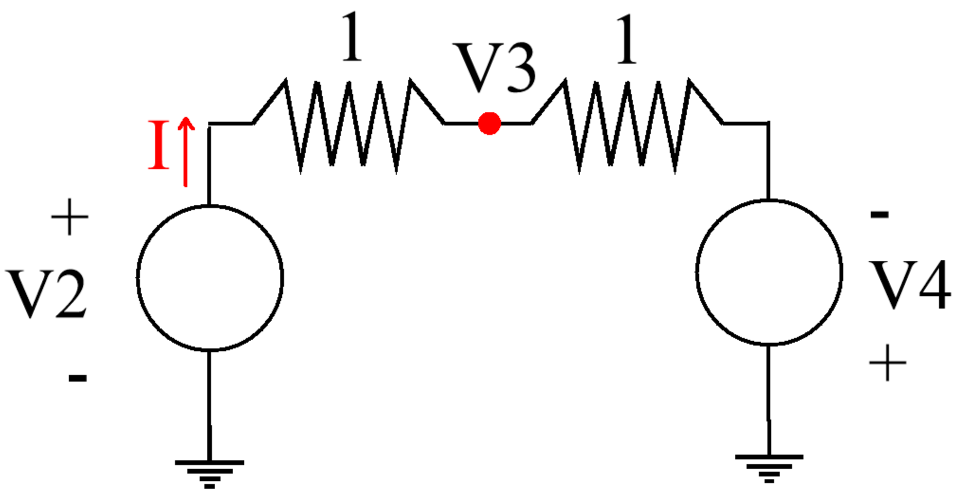
$$V_4 = k \times V_3 \qquad (5)$$

where k is some number that has no units.

$$V_3 = \frac{V_4}{k} \qquad (5')$$

is the same thing rearranged.

While V_2 can be any value we want, V_4 depends on V_3 to set its value.



As before, I can calculate my current and then find V_3.

$$I = \frac{V_2 + V_4}{1\Omega + 1\Omega} \qquad (6)$$

$$V_3 = V_2 - (I) \times 1\Omega \quad (7)$$

Combining 6 and 7 I get

$$V_3 = V_2 - \frac{V_2 + V_4}{2}$$

Which gives me

$$V_3 = \frac{V_2}{2} - \frac{V_4}{2} \qquad (7)$$

Now I will drop in 5'

$$V_3 = \frac{V_4}{k} \qquad (5')$$

to get

$$\frac{V_4}{k} = \frac{V_2}{2} - \frac{V_4}{2} \qquad (8)$$

$$\frac{V_4}{k} = \frac{V_2}{2} - \frac{V_4}{2} \qquad (8)$$

Which can be rearranged:

$$V_4 \times \left(\frac{1}{k} + \frac{1}{2}\right) = \frac{V_2}{2} \qquad (9)$$

$$V_4 = \frac{\left(\frac{V_2}{2}\right)}{\left(\frac{1}{k}+\frac{1}{2}\right)} \qquad (10)$$

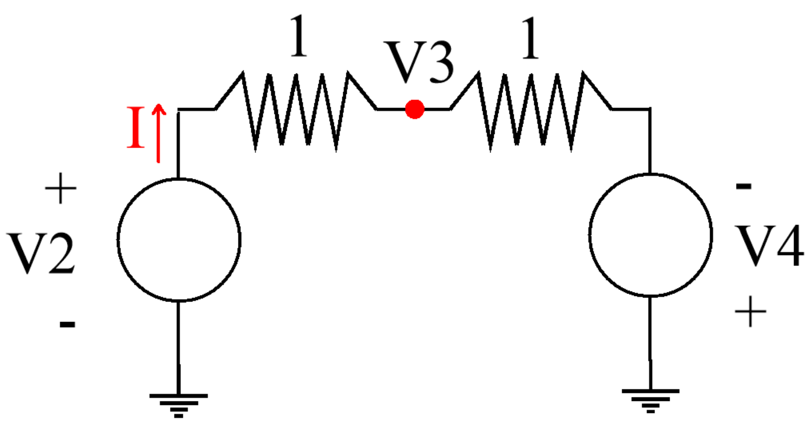
$$V_4 = \frac{\left(\frac{V_2}{2}\right)}{\left(\frac{1}{k}+\frac{1}{2}\right)} \qquad (10)$$

Look what happens as I increase the value of *k*. As *k* gets larger, $\frac{1}{k}$ gets smaller. At some point, $\frac{1}{k}$ is so much smaller than ½ that I can, for practical purposes, ignore it. Then I can say

$$V_4 \approx \frac{\left(\frac{V_2}{2}\right)}{\left(0+\frac{1}{2}\right)} \qquad (11)$$

$$V_4 \approx V_2 \qquad (12)$$

Where ≈ is read as "approximately equal to."



This says that by setting

$$V_3 = \frac{V_4}{k} \qquad (5')$$

and making *k* very large, we cause V_4 to approximately equal V_2. As *k* gets larger, this approximation gets better.

For typical opamps, *k* is at least 100,000 so we can say that V_4 equals V_2. We created a "unity gain inverting amplifier." If V_2 is 1v, V_4 is -1v.

How accurate is the circuit? Say V_2 equals 1volt and k is 100,000. Then, using equation 10,

$$V_4 = \frac{\left(\frac{V_2}{2}\right)}{\left(\frac{1}{k}+\frac{1}{2}\right)} \qquad (10)$$

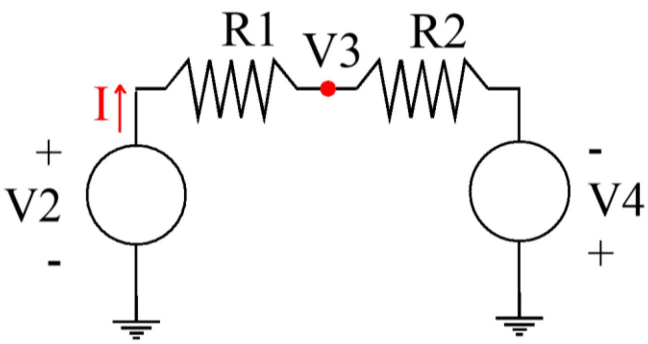
we know

$$V_4 = \frac{\left(\frac{1\ volt}{2}\right)}{\left(\frac{1}{100{,}000}+\frac{1}{2}\right)} = 0.99998\ volts$$

Not bad at all! Do understand that we are assuming an ideal opamp. Real opamps have other imperfections, which I will deal with later.

The General Case

I will next go through this logic again but for the general case.



$$I = \frac{V_2 + V_4}{R_1 + R_2} \qquad (13)$$

$$V_3 = V_2 - (R_1 \times I) \qquad (14)$$

$$V_3 = V_2 - \left\{(R_1) \times \left(\frac{V_2+V_4}{R_1+R_2}\right)\right\} \qquad (15)$$

This is starting to look messy, so let me define

$$H = \frac{R_1}{R_1+R_2} \qquad (16)$$

Then 15 becomes

$$V_3 = V_2 - \{(H) \times (V_2 + V_4)\} \qquad (16)$$

Combining like terms, I get

$$V_3 = \{(V_2) \times (1 - H)\} - \{(V_4 \times H)\} \qquad (17)$$

Again, use equation 5': $V_3 = \frac{V_4}{k}$ and I get

$$\frac{V_4}{k} = \{(V_2) \times (1 - H)\} - \{(V_4 \times H)\} \qquad (17)$$

Grouping like terms and rearranging,

$$V_4 = \frac{1-H}{\left(\frac{1}{k}+H\right)} \times V_2 \qquad (18)$$

After letting *k* get very large, I can say

$$V_4 \approx \frac{1-H}{(H)} \times V_2 \qquad (19)$$

$$V_4 \approx \frac{1-H}{(H)} \times V_2 \qquad (19)$$

$$V_4 \approx \left(\frac{1}{H} - 1\right) \times V_2 \qquad (19)$$

Where

$$H = \frac{R_1}{R_1+R_2} \qquad (16)$$

Therefore, I can say

$$V_4 \approx \left(\frac{R_1 + R_2}{R_1} - 1\right) \times V_2$$

or

$$V_4 \approx \left(\frac{R_2}{R_1}\right) \times V_2 \qquad (20)$$

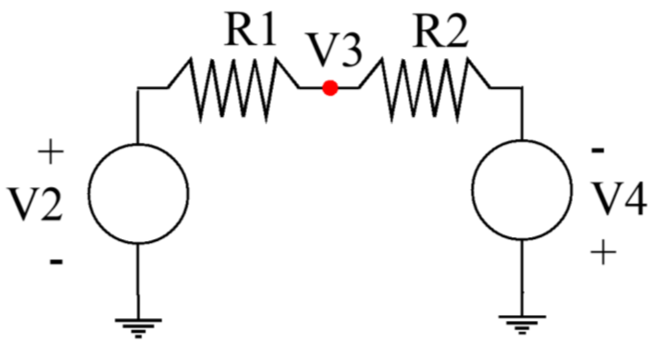
As a check, I will set R_1 and R_2 equal to 1Ω.

$$V_4 \approx \left(\frac{1}{1}\right) \times V_2$$

$$V_4 \approx (1) \times V_2$$

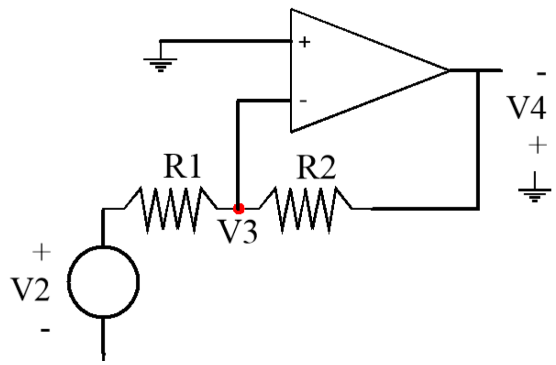
which matches equation 12.

Back to Our Ideal Operational Amplifier



$$V_4 \approx \left(\frac{R_2}{R_1}\right) \times V_2 \qquad (20)$$

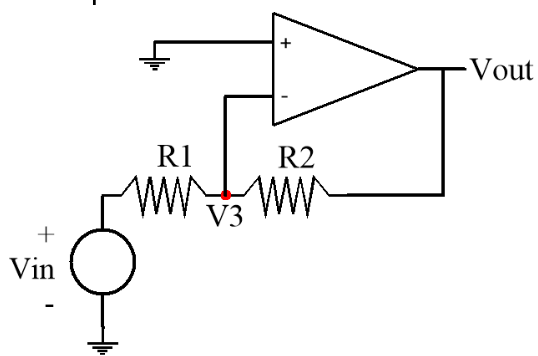
It is time to return to the beginning. If I ground the positive input of my ideal opamp and define the negative input as V_3, then my output is V_4. Note that our V_4 voltage source is hidden inside the opamp symbol.



It is traditional to talk about V_{in} and V_{out}.

$$V_{in} = V_2$$

$$V_{out} = -V_4$$



so we get

$$V_{out} \approx -\{\left(\frac{R_2}{R_1}\right) \times V_{in}\} \qquad (21)$$

or simply

$$V_{out} \approx -\frac{R_2}{R_1} V_{in} \qquad (22)$$

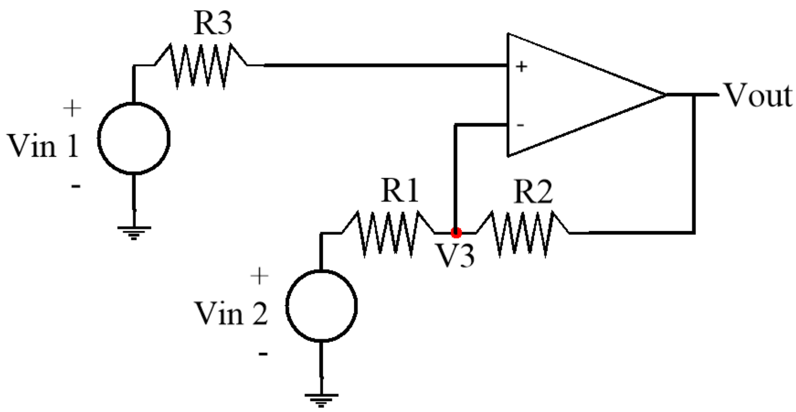
With R_1 equal to R_2,

$$V_{out} \approx -V_{in} \qquad (23)$$

$\frac{V_{out}}{V_{in}}$ is called our Closed Loop Gain. In this case, it is approximately -1.

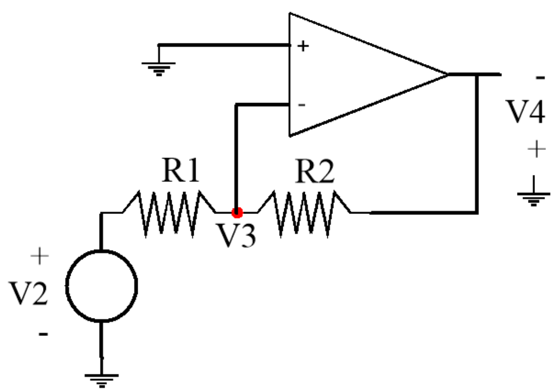
A Two Input Operational Amplifier circuit

If you are still with me, it is time to take this up a notch.

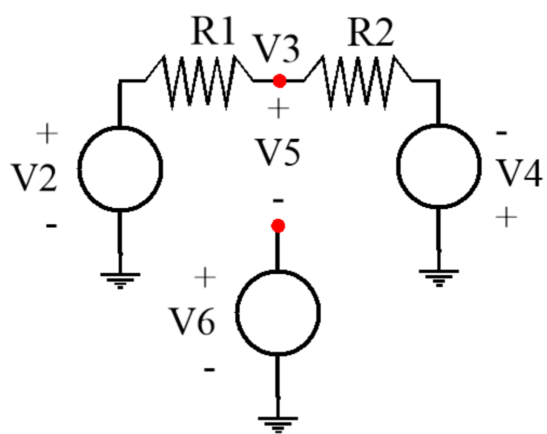


This circuit has two inputs, $V_{in\ 1}$ and $V_{in\ 2}$. They somehow combine to drive V_{out}.

Recall that



V_3 is relative to ground but also that the positive input of the opamp was grounded.



Returning to our last model, I have defined a new voltage source called V_6 and measure a voltage, V_5 between it and the voltage at node 3.

This time, I define

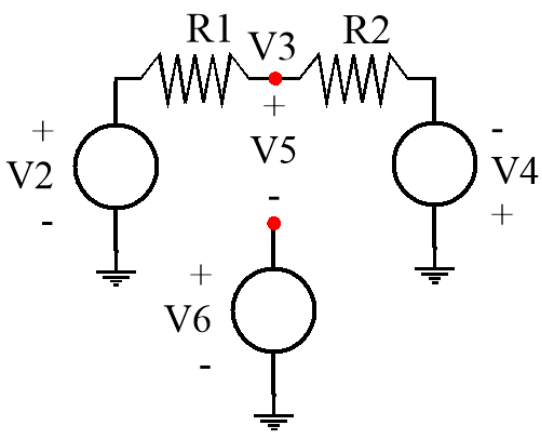
$$V_4 = k \times V_5 \qquad (24)$$

which I can also write as

$$V_5 = \frac{V_4}{k} \qquad (25)$$

I could run through the same logic as before to generate an equation but, instead, will use a different approach.

$$V_5 = \frac{V_4}{k} \qquad (25)$$



As k gets "large", equation 25 tells us that for any "reasonable" value of V_4, V_5 must get "very small." These are vague terms so I will use a concrete example.

Say the opamp is powered from + and – 10 volts. Then V_4 can't be any larger than 10 volts. If k is 100,000, then V_5 can't be any larger than $\frac{10\ volts}{100{,}000}$ which is 0.0001 volts. If k was infinite, V_5 would be 0 volts.

Note that V_3 equals V_6 plus V_5. Say V_2 is 2 volts, V_6 is 1 volt, and R_1 is 1 ohm. V_5, at 0.0001 volts, means V_3 equals 1.0001 volts. The current through R_1 is

$$I = \frac{V_2 - V_3}{R_1}$$

$$I = \frac{2\ volts - \ 1.001\ volts}{1\Omega}$$

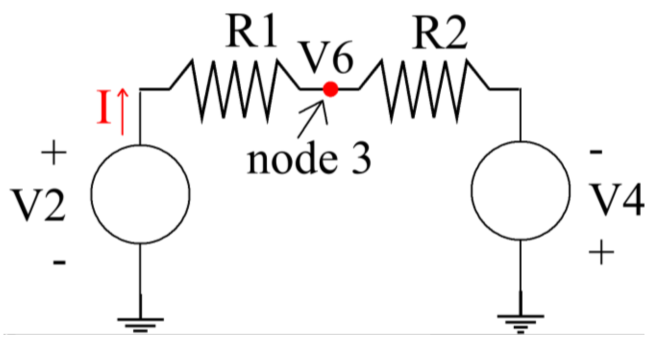
$$I = \ 0.999\ amperes$$

If V_5 was 0 volts V_3 would equal 1.0000 volts

$$I = \frac{2\ volts - \ 1.000\ volts}{1\Omega}$$

$$I = 1.000\ amperes$$

Therefore, if k went from 100,000 to infinite, our current would change from 0.999 amperes to 1.000 amperes. In other words, as long as k is "large", it has little effect on the circuit. It is common to see ≈ replace by = since our approximation is so close to exact. I will continue to use ≈ to remind you that it depends on the magnitude of k.



Accepting that V_5 is essentially zero, V_3 must be essentially equal to V_6. We can then calculate the current, *I*.

What is strange here is that *I* set the voltage V_6 without forcing node 3 directly with a voltage source. Three is no direct connection from V_6 to node 3. V_6 is indirectly set by V_2, V_4, R_1, and R_2. This can be a difficult concept to grasp but is part of what I call the magic of feedback. As *k* gets large, we have a *virtual short* between nodes 6 and 3 yet no current passes.

I will retreat to the equations to figure this out.

$$I = \frac{V_2 - V_6}{R_1} \qquad (26)$$

Notice that the current, *I*, must all flow through R_2 because nothing else is connected to node 6. I can say

$$V_6 - \{R_2 \times I\} \approx -V_4 \qquad (27)$$

I used ≈ because node 3 is approximately equal to V_6.

Combining 26 and 27 I get

$$V_6 - \left\{R_2 \times \frac{V_2 - V_6}{R_1}\right\} \approx -V_4$$

Rearranging terms I get

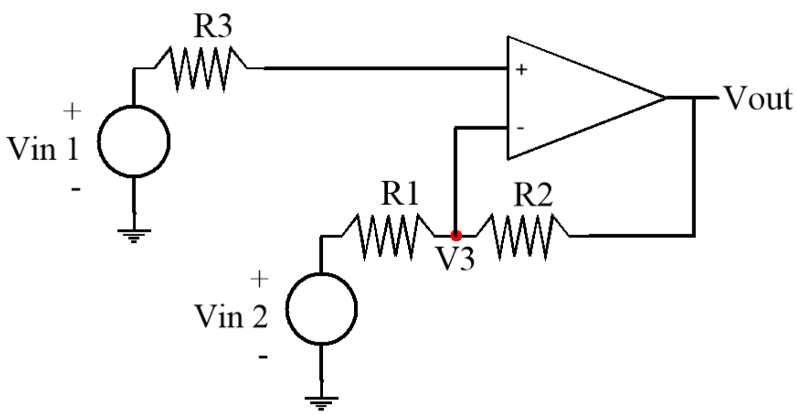
$$V_4 \approx \frac{R_2}{R_1} V_2 - \left(1 + \frac{R_2}{R_1}\right) V_6 \qquad (28)$$

$$V_4 \approx \frac{R_2}{R_1} V_2 - \left(1 + \frac{R_2}{R_1}\right) V_6 \qquad (28)$$

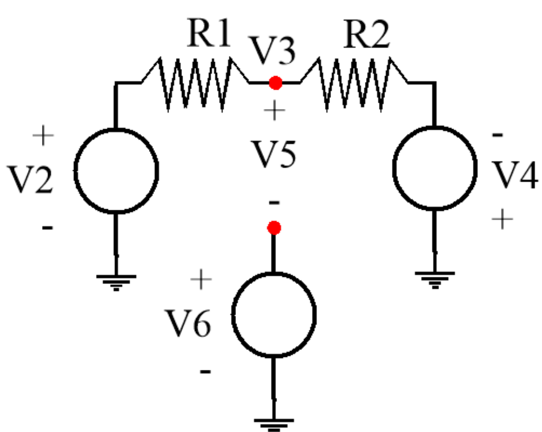
If I set V_6 to zero, equation 28 becomes equation 20.

$$V_4 \approx \left(\frac{R_2}{R_1}\right) \times V_2 \qquad (20)$$

That's comforting.



Returning to our opamp symbol, understand that our ideal opamp's positive and negative inputs only measure voltage. They do not draw any current. This means there is no current through R_3, so no voltage drop across it either. $V_{in\ 1}$ is, therefore, equal to the positive input voltage.



In our model, the positive input voltage is represented by V_6. I can say

$$V_{in\ 1} = V_6 \qquad (29)$$

We know by comparing the opamp circuit with our model that

$$V_{out} = -V_4 \qquad (30)$$

$$V_{in\ 2} = V_2 \qquad (31)$$

$$V_{in\ 1} = V_6 \quad (29)$$

$$V_{out} = -V_4 \quad (30)$$

$$V_{in\ 2} = V_2 \quad (31)$$

$$V_4 \approx \frac{R_2}{R_1} V_2 - \left(1 + \frac{R_2}{R_1}\right) V_6 \quad (28)$$

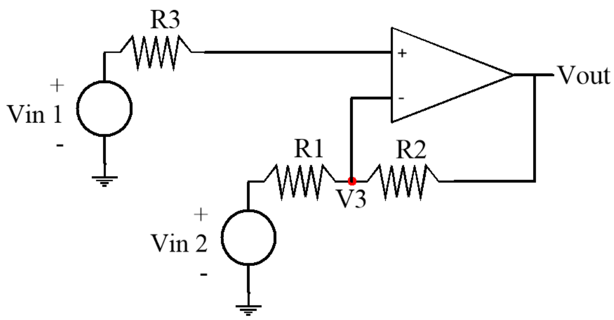
Combining equations 29, 30, 31, and 28 yields

$$-V_{out} \approx \frac{R_2}{R_1} V_{in\ 2} - \left(1 + \frac{R_2}{R_1}\right) V_{in\ 1} \quad (32)$$

or

$$V_{out} \approx \left(1 + \frac{R_2}{R_1}\right) V_{in\ 1} - \frac{R_2}{R_1} V_{in\ 2} \quad (33)$$

Let’s try out a few cases.



$$V_{out} \approx \left(1+\frac{R_2}{R_1}\right)V_{in\,1} - \frac{R_2}{R_1}V_{in\,2} \qquad (33)$$

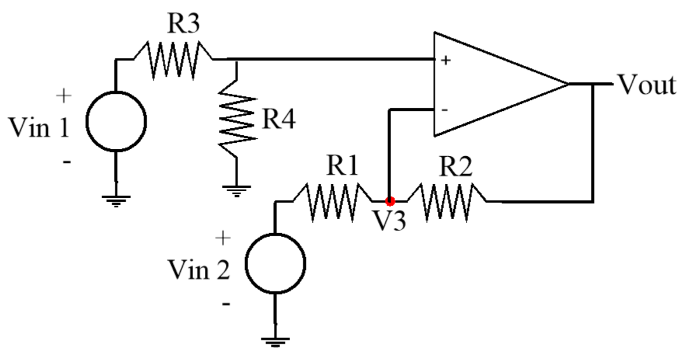
Say $V_{in\,2}$ is zero. We have a noninverting amplifier:

$$V_{out} \approx \left(1+\frac{R_2}{R_1}\right)V_{in\,1} \qquad (34)$$

If $V_{in\,1}$ is zero, we have our original configuration with

$$V_{out} \approx -\frac{R_2}{R_1}V_{in\,2} \qquad (20)$$

Notice that no values of R_1 and R_2 enable me to achieve $V_{in\,1} - V_{in\,2}$.



However, with one more resistor, we can do it. Set all four resistors to the same value, such as 1.

Then we would have

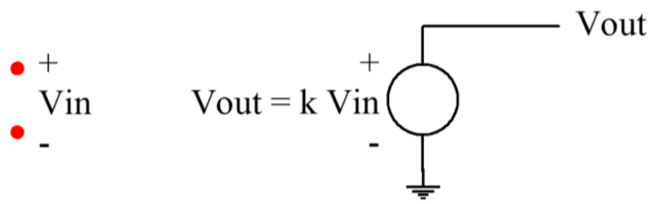
$$V_{out} \approx \{\left(1+\frac{R_2}{R_1}\right) \times \left(\frac{R_4}{R_4+R_3}\right)\}V_{in\,1} - \frac{R_2}{R_1}V_{in\,2} \qquad (35)$$

$$V_{out} \approx \{\left(1+\frac{1}{1}\right) \times \left(\frac{1}{1+1}\right)\}V_{in\,1} - \frac{1}{1}V_{in\,2}$$

$$V_{out} \approx V_{in\,1} - V_{in\,2} \qquad (36)$$

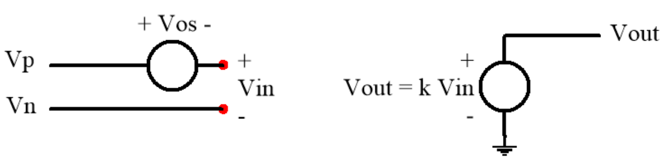
Non-ideal Operational Amplifier Parameters

Alas, nothing is perfect. A real opamp has many parameters that define its deviation from the ideal. They also make the circuit more complicated. Once you understand these parameters, you can find ways to minimize their effect.



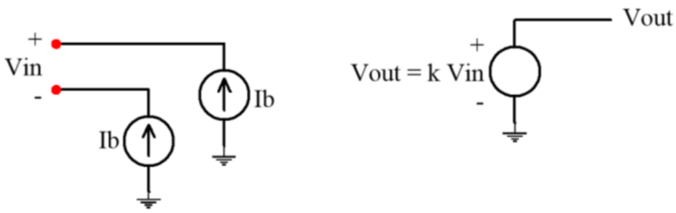
I can build on my ideal opamp model to include all of the parameters in a real opamp's spec sheet.

Input Offset Voltage



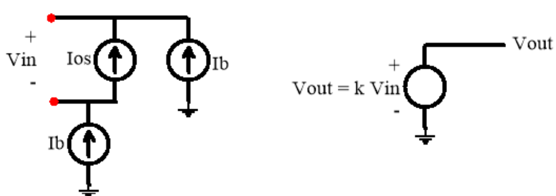
Input Offset voltage, V_{os}, can be modeled by a DC voltage source in series with one of the inputs. Since V_{os} can be of either polarity, it doesn't matter where I put the positive terminal.

Input Bias Current



Typically, a current flows out of the input nodes if PNP transistors are in the input stage. It flows in for NPN transistors[3]. These currents are independent of input voltage so I can model them as current sources.

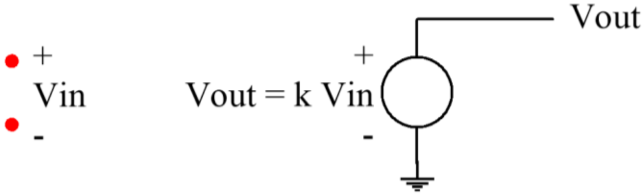
Input Offset Current



The two current sources are not identical; there is a difference between them that we call the Input Offset current, I_{os}. As with V_{os}, it can be of either polarity.

[3] I noticed that some opamps have input bias current that can be positive or negative and is very small. This implies that they are trying to internally cancel the current. Externally, you see the error current.

Gain



I have modeled the gain of the opamp as *k*. This is typically referred to as the open loop gain in the spec sheet. Recall, from page 11, that the closed loop gain is the circuit's output voltage divided by its input voltage[4].

If you look at the LM358's spec sheet, you will see open loop gain designated as A_{OL}. The minimum is 70 volts/mv, which is another way of saying 70,000. The typical value is 140 volts/mv or 140,000. Don't let this confuse you. The opamp will not output 70, let alone 140 volts, when you input 1 millivolt. More on this when I talk about output voltage.

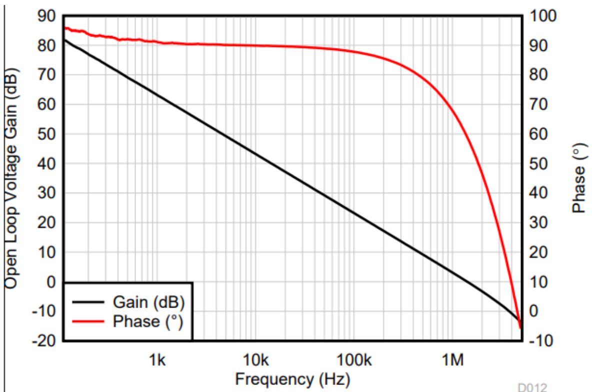


Figure 7-5. Open-Loop Gain and Phase vs Frequency

Ah, but there is more, and it may seem strange. This gain is only at DC. It falls off as frequency rises. This behavior ensures that the opamp doesn't become unstable as you put your resistors around it[5].

Recall that *k* can vary a lot and have only a small effect on the overall circuit's operation. So, while *k* is falling as we raise frequency, the overall circuit's gain is essentially constant until *k* is no longer "large."

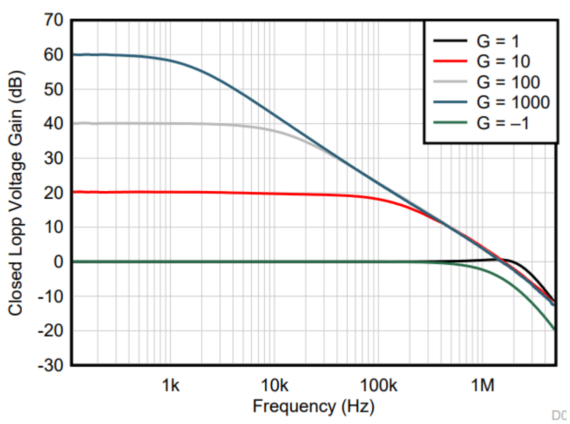


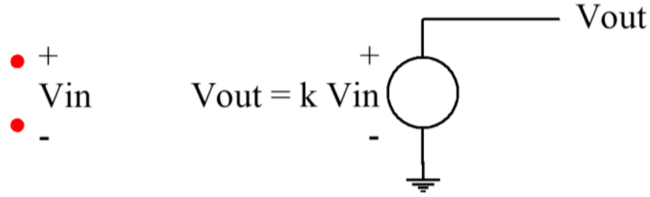
Figure 7-6. Closed-Loop Gain vs Frequency

For example, if my closed loop gain, G, is 1 for the LM358, all is fine until about 1 MHz. If the closed loop gain is 100, I'm only good to about 10 KHz.

[4] If the output was voltage and the input was current, we talk about transimpedance. If the output was current and the input was voltage, we talk about transadmittance.

[5] Both the gain and time delay (phase) are carefully controlled such that the circuit never reaches positive feedback at any frequency. That would cause the amplifier to turn into an ossilator.

Input Voltage Range



The input nodes must stay within specified boundaries for the opamp to process their voltages. This is called the Common Mode Voltage Range. For the LM358, this input can go as low as the negative supply rail. They can go as high as within 2 volts of the positive supply rail.

For example, if the opamp is powered from +5V and ground, the inputs can be as low as 0 and as high as 5 – 2 = 3 volts.

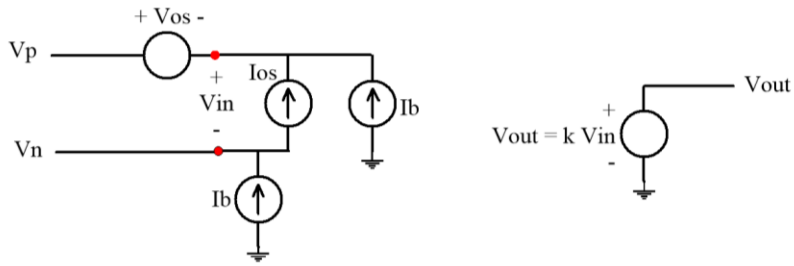
Ideally, the opamp only looks at $V_{in,}$ but the output is slightly sensitive to input movement relative to ground. This is the Common-mode rejection ratio.

Output

Our ideal output voltage does have limited capabilities. It can swing just so close to the positive rail and just so close to the negative rail. For the LM328 powered by 30 volts and ground, it can output up to 26 volts and down to 20 millivolts.

This output voltage also has a limit on how much current it can source and sink. For the LM328, it can typically source 30 milliamps and sink 20 milliamps. The fine print says that when you get down to an output voltage of 20 millivolts, it can only sink a few microamps.

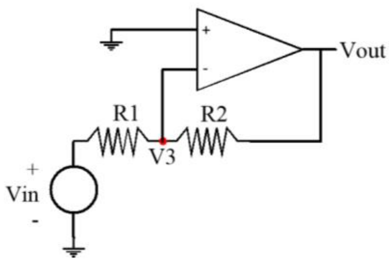
A More Accurate Model



This is not a complete model since many of the opamp's limitations are described in words and graphs and not with circuit elements.

Note that I have taken our ideal opamp and hung current and voltage sources on it to represent non-ideal parameters found in the spec sheet. On the one hand, this enables us to use circuit analysis techniques previously employed. On the other hand, the circuit, and corresponding math, get more complex.

Using The Model

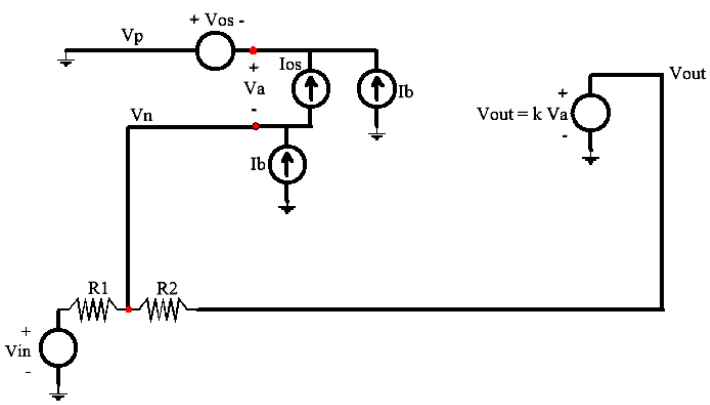


If the opamp was ideal, we know that

$$V_{out} \approx -\frac{R_2}{R_1}V_{in} \qquad (22)$$

This equation is about to get more complex as we drop in our more accurate model.

I have renamed V_{in} in the model to V_a to avoid a conflict with the external input voltage. V_3 is V_n.



$I_b - I_{os}$ flows out of V_n. This causes a new current to flow in R_2, adding to V_{in}'s current.

A possibly surprising result is that $I_b - I_{os}$ effectively does not flow in R_1 even though it is connected to node *n*. This is because of how the feedback works.

V_a now contains V_{os} and the V_n, which used to be V_3.

Analysis of this circuit gets a bit messy so I will skip the intermediate steps.

$$V_a = -V_{os} - V_n \qquad (37)$$

Summing the current going into node *n* I get

$$\frac{V_{in}-V_n}{R_1} + I_b - I_{os} + \frac{V_{out}- V_n}{R_2} = 0 \qquad (38)$$

$$V_{out} = k \times V_a \qquad (39)$$

$$V_a = -V_{os} - V_n \tag{37}$$

$$\frac{V_{in} - V_n}{R_1} + I_b - I_{os} + \frac{V_{out} - V_n}{R_2} = 0 \tag{38}$$

$$V_{out} = k \times V_a \tag{39}$$

Combining 37, 38, and 39 and let *k* be large gives us

$$V_{out} \approx -\frac{R_2}{R_1} V_{in} + V_{error} \tag{40}$$

where

$$V_{error} \approx \left\{\frac{R_2}{R_1} + 1\right\} V_{os} - \{R_2\}\{I_b - I_{os}\} \tag{41}$$

Equation 40, with V_{error} equal to zero, is our old friend, equation 22.

V_{error} has a few lessons in it. First, let's look at offset voltage and then the bias and offset currents.

As $\frac{R_2}{R_1}$ gets larger, we amplify V_{in} more, but that also amplifies V_{os}. For example, with a $\frac{R_2}{R_1}$ = 1, V_{out} contains $\{1 + 1\}V_{os} = 2 \times V_{os}$.

For the LM328, V_{os} can have a magnitude as large as 9 mV. It appears at the output as $\{1 + 1\}9\ mV =$ 18 mV. Since V_{os} can be positive or negative, we can expect it to cause an error between -18 mV to +18 mV at the output.

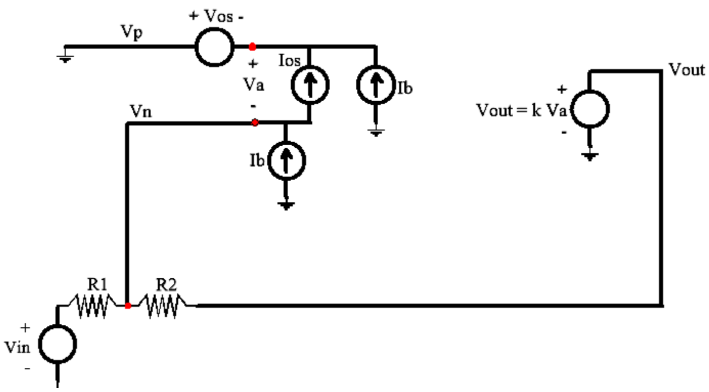
If our output is 10 volts, this doesn't hurt much. But if our ideal output is 18 mV, this error causes the output to be between zero and 36 mV. In other words, the circuit is useless.

Look what happens when I increase amplification. If $\frac{R_2}{R_1}$ = 10, V_{out} contains $11 \times V_{os}$. It appears at the output as ± 99 mV.

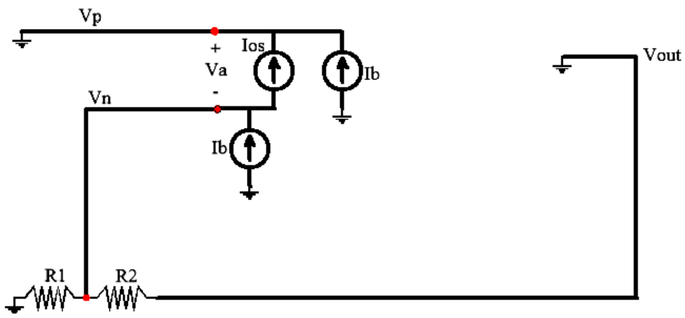
$$V_{out} \approx -\frac{R_2}{R_1} V_{in} + V_{error} \qquad (40)$$

where

$$V_{error} \approx \left\{\frac{R_2}{R_1} + 1\right\} V_{os} - \{R_2\}\{I_b - I_{os}\} \qquad (41)$$



I_b and I_{os} hold a surprise.



If my circuit did not contain the opamp, I would write an equation for the current going into node *n*:

$$I_b - I_{os} + \frac{0 - V_n}{R_1} + \frac{0 - V_n}{R_2} = 0 \qquad (42)$$

Solving for V_n, I get

$$V_n = \left(\frac{1}{\left[\frac{1}{R_1} + \frac{1}{R_2}\right]}\right) \times (I_b - I_{os}) \qquad (43)$$

You may recognize the resistor part of this equation as R_1 in parallel with R_2. Equation 43 says that $I_b - I_{os}$ flow into node *n* and through R_1 and R_2 to generate V_n.

$$V_{error} \approx \left\{\frac{R_2}{R_1} + 1\right\} V_{os} - \{R_2\}\{I_b - I_{os}\} \qquad (41)$$

Now, compare this to equation 41. Notice that R_1 is missing from the term involving I_b and I_{os}. Is this an algebra error? I don't think so. If you derive 41, you will see where R_1 is divided by *k*. As *k* gets large, R_1 vanishes.

$$V_{out} \approx -\frac{R_2}{R_1}V_{in} + V_{error} \tag{40}$$

where

$$V_{error} \approx \left\{\frac{R_2}{R_1} + 1\right\}V_{os} - \{R_2\}\{I_b - I_{os}\} \tag{41}$$

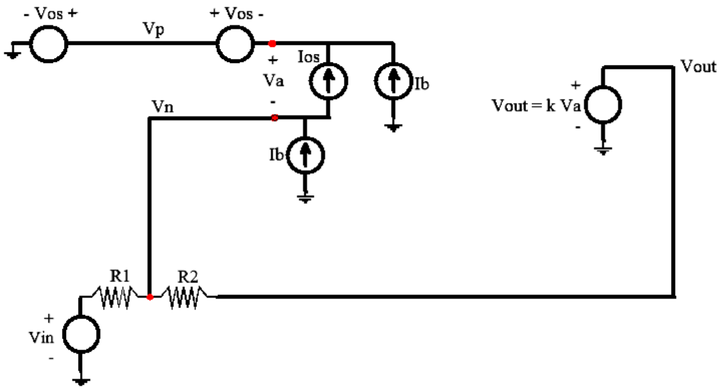
As R_2 gets larger, the effect of $I_b - I_{os}$ gets larger. For the LM328 opamp, I_b can be as large as 0.5 µA. I_{os} can be as large as -0.15 µA. The worst-case current would then be 0.5 – (-0.15) = 0.65 µA.

Say R_2 was 10K, then $\{R_2\}\{I_b - I_{os}\}$, in the worst case, the result would be 6.5 mV. If R_2 was 100K, it would generate 65 mV.

The take-home message is that designing a high gain DC amplifier is hard to do. These non-ideal parameters have a significant impact on the output voltage.

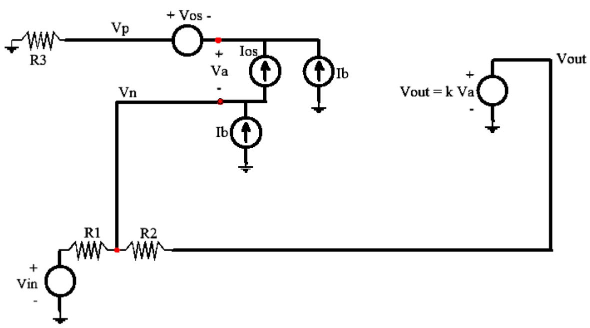
Reducing Non-Ideal Effects

We can add circuit elements that reduce V_{error}.



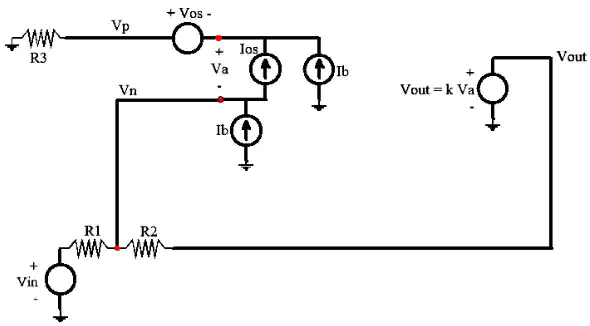
If we knew V_{os}, we could put a voltage source in series with an equal but opposite value. This is identical to setting V_{os} to zero.

There are two challenges here. First, you must know V_{os}. Second, building that ideal voltage source will take another circuit.



I can cancel the effect of I_b without knowing its value by adding resistance on the V_p side. Recall that I_b on the V_n side sees only R_2.

If I put a resistor, R_3, equal to R_2, between V_p and ground, I_b will raise the voltage on the positive and negative inputs by the same amount. This means V_a will have no net change due to I_b.



I_{os} and V_{os} vary from device to device plus drift as a function of temperature, as noted in the spec sheet. This makes driving V_{error} all the way to zero near impossible. However, adding R_3 is low cost and effective, so is commonly done.

Acknowledgments

Thanks to my tutoring students at Glendale Community College for trying this out. Thanks to Andrew Ayers for suggesting I add non-ideal elements. And thanks to "buffalojohn" for challenging me on my focus and clarity.

I welcome your comments and questions.

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