The Theory and Practice of Using a Sine Bar, version 2

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The Quick Answer

If you just want to set an angle with a sine bar and stack of blocks, then take the sine of the desired angle on your calculator and multiply the result by the distance between the centers of the cylinders in the sine bar. Assemble a stack of blocks equal to this value and put it under one of the cylinders.

It is absolutely essential that all contact surfaces be clean. Bits of swarf can easily get between blocks and give unexpected results. Start by cleaning your hands and only then clean all surfaces with a solvent that does not leave a residue.

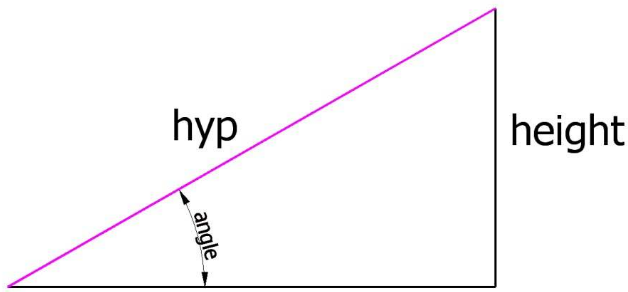
The Long Answer

Given a sine bar and a set of spacer blocks or Jo blocks, you can set a wide range of angles with extremely high accuracy. Oh yes, and you will also need a calculator able to handle the sine and arcsine function.

The minimum angle you can set will equal the smallest step size in you block set. The maximum angle will be less than 90°. At some point the sine bar will fall over.

I will first talk about the theory behind a sine bar, then the practical side. The last section gets kind of deep as I talk about worst case error due to imperfections in the sine bar and spacer or Jo blocks. I can’t claim “extremely high accuracy” and then ignore the limitations on this accuracy.

The Theory



Given a right triangle, we can relate the hypotenuse, height, and angle with the formula

$$angle = \sin^{-1}\left(\frac{height}{hyp}\right) \qquad \text{(equation 1)}$$

Where $\sin^{-1}$ is the arcsine. Some calculators will display the angle in degrees or radians. Be sure you are in degrees.

Say we have a "hyp" equal to 5 and a height equal to 1. Then we can calculate the angle

$$angle = \sin^{-1}\left(\frac{1}{5}\right) = 11.537°$$

I have chosen to round the answer to 3 places to the right of the decimal point. How much to round the answer will be addressed in the last section where I talk about errors.

I can also go the other way. Say I want to set a given angle using a sine bar and a set of spacer blocks. Then I will need

$$\text{height} = \text{sine bar length x} \sin(angle) \qquad \text{(equation 2)}$$

For example, say I want an angle of 12° and have a sine bar 5″ long:

Height = (5″) x sin (12°)
Height = (5″) x 0.2079
Height = 1.03956″

Here is where we crash into reality. My cheap little set of spacer blocks is only able to give me heights to the nearest 0.001". So how bad is that? I can set a height of 1.040″. If I plug this height and my 5″ sine bar length into equation 1 I will get

$$angle = \sin^{-1}\left(\frac{1.040}{5}\right) = 12.005°$$

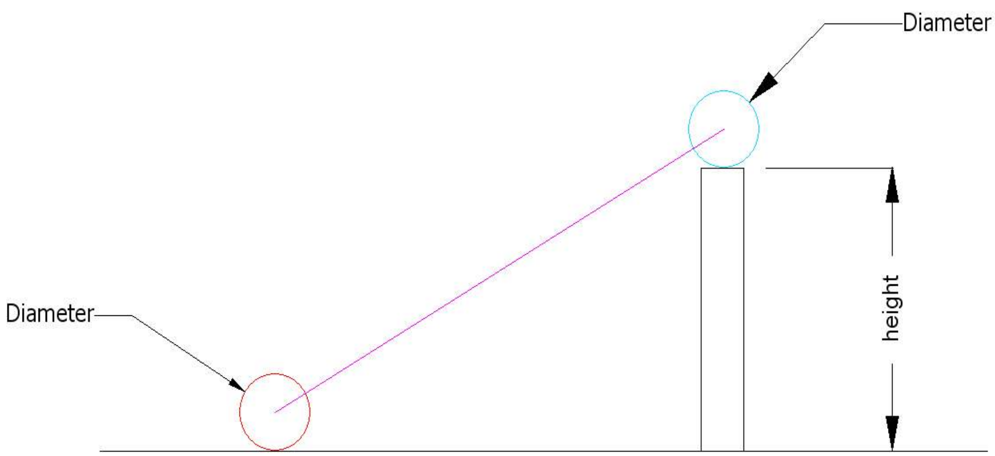
OK, maybe I should not feel so bad about the short comings of my spacer block set. To give you an idea of what and error of 0.005° means, consider sighting a building with the sight set at 12° and then at 12.005°. If you are looking one mile away, then this angular difference of 0.005° means that you have moved 5.4″ up the building. That isn't much given you are one mile away.

For more on using spacer blocks, see

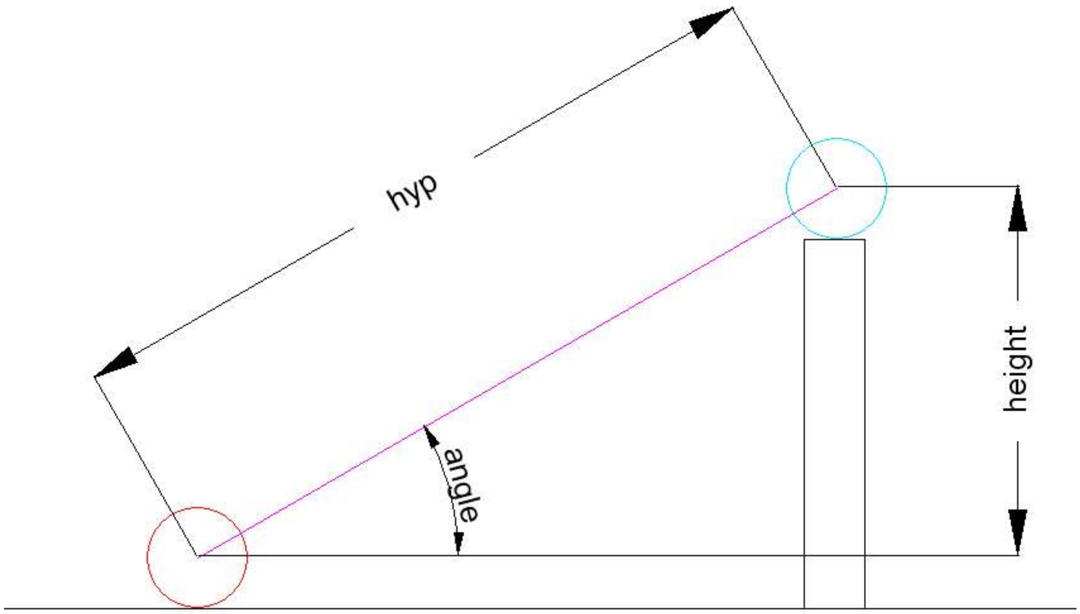
http://rick.sparber.org/sbk.pdf

The Practice

First consider the case of two cylinders of exactly the same diameter. I have drawn a line between their centers. Since they have the same diameters, they must also have the same radius. With the two cylinders sitting on a perfectly flat surface, I can say that the line between centers is exactly parallel to my flat surface and is above it by the radius of either cylinder.



Look what happens if I put one of the cylinders on a block of known height. The line between centers will now be at an angle with respect to the horizontal.



Here is where having the two cylinders with the exact same radius comes in handy. The lower end of the sloped line is one radius above my flat surface. The upper end is a one radius above the top of our block. So if I draw a horizontal line from the lower cylinder's center over to the block, I will be one radius above the flat surface which is supporting my block.

Resting on the top of the block is my second cylinder with its center one radius above the top of the block. The height of my block is simply shifted up by the radius. My hyp, height, and angle are again related by the formula

$$angle = \sin^{-1}\left(\frac{height}{hyp}\right) \qquad \text{(equation 1)}$$

The diameter of the two identical cylinders cancel each other so don't appear in the formula.

One more observation: note that the height is the ***difference*** between the centers or the bottoms of each cylinder. I could put the lower cylinder at a height of 4' above the floor and the upper cylinder 4' 1″ above the floor. The height as defined above would be 4' 1″ - 4'= 1″. If the hyp was 5″, then I would get

$$angle = \sin^{-1}\left(\frac{1}{5}\right) = 11.537°$$

Sine bars are specified by the distance between the centers so I am using a 5 inch sine bar here.

Now let's bring back the spacer blocks.

Say I want to set the smallest possible angle using my 5 inch sine bar. I have a spacer block 0.106″ thick and another one 0.107″ thick. If I put the 0.106″ thick block under one cylinder and the 0.107″ thick block under the other cylinder, then my "height" will be 0.107″ - 0.106″ = 0.001". My "hyp" will be 5″.

$$angle = \sin^{-1}\left(\frac{0.001}{5}\right) = 0.011°$$

where I have rounded the angle to 2 places to the right of the decimal point.

Error Analysis of the Smallest Angle

So far I have assumed that my sine bar and spacer blocks are perfect. Alas, they are not. Sorry guys, but if we want to talk about precision, we must talk about error and that means a bit of math and logic.

My spacer blocks are good to ± 0.0001″ each so, worst case, my height would be 0.001 ±0.0002″ because I have two blocks defining the height. As mentioned above, the smallest step I can make with my spacer blocks is 0.001″.

This means that my angle will also have an uncertainty. For now, assume my sine bar is exactly 5″ center to center.

If my height is really 0.001 + 0.0002″, then my angle will be 0.014°. If my height is really 0.001 – 0.0002″, then my angle will be 0.009°. In other words, my angle will be somewhere between 0.009° and 0.014° or I can say 0.011 ± 0.003°.

This is the minimum height I can achieve with my spacer blocks. Look what I get if I use my 1″ block under one cylinder and nothing under the other cylinder. The height will be 1 ± 0.0001″. This means that my angle will be between 11.536° and 11.538° or I can say 11.537 ± 0.001°.

Now that is odd. When my height was 0.001″, my angular error was ± 0.003° while with a height of 1″ my angular error is one third of it. The reason for this change in error is hidden in the error found in the spacer blocks. All of the blocks are ± 0.0001″ each. With my height of only 0.001″ and using two blocks, my height error is ± 0.0003″ or 30%. But with my height of 1″ and using a single block, my height error is ± 0.0001″ or 0.01%.

The bottom line is that you can calculate the angular error only by first finding the error in the height and then using the formula to find the minimum and maximum angle.

Angular Error Analysis in the General Case

Adding in the error associated with the sine bar does complicate things a bit but is necessary in order to find the total error.

Recall that

$$angle = \sin^{-1}\left(\frac{height}{hyp}\right) \qquad \text{(equation 1)}$$

Say I want to calculate the ***maximum*** angle. I want to find the ***maximum*** height due to the spacer block error. I want to find the ***minimum*** hyp which is the error in the sine bar. These choices will maximize the ratio of height to hyp.

For example, say my height is 1 ± 0.0001″ and my sine bar is 5 ± 0.0002″. My maximum angle will be

$$angle = \sin^{-1}\left(\frac{1+0.0001}{5-0.0002}\right) = 11.539°$$

To calculate the ***minimum*** angle, I want to find the ***minimum*** height due to the spacer block error and ***maximum*** hyp due to the sine bar error. This gives me

$$angle = \sin^{-1}\left(\frac{1-0.0001}{5+0.0002}\right) = 11.535°$$

My worst case error is therefore ± 0.002° with an ideal value of 11.537°.

Now, for anything I do around my shop, this error is laughably small so I usually ignore it.

Height/Angle Error Analysis in the General Case

We will start by using

$$\text{height} = \text{sine bar length x} \sin(angle) \qquad \text{(equation 2)}$$

The first step is to find the ideal height given the nominal length of our sine bar and the desired angle. Given the limitations of our spacer or Jo block set, we must approximate the height. Take this approximation and plug it into

$$angle = \sin^{-1}\left(\frac{height}{hyp}\right) \qquad \text{(equation 1)}$$

If you want to know the error bounds of this angle, then use the procedure mentioned on the last page.

It is now time for a full example.

I have a sine bar 3 ± 0.0002″ long and I want an angle of 22°.

My ideal height is calculated from equation 2

$$\text{height} = \text{sine bar length x} \sin(angle)$$
$$\text{height} = 3'' \text{ x} \sin(22°)$$
$$\text{height} = 3'' \text{ x } 0.37461$$
$$\text{height} = 1.1238''$$

The closest I can get with my spacer blocks is 1.124″ which would be formed from a 0.104″, 0.120″, and a 0.900″ block. Given that I have 3 blocks, my maximum worst case error would be ± 0.0003″.

I now go to equation 1 to find the error.

$$angle = \sin^{-1}\left(\frac{height}{hyp}\right) \qquad \text{(equation 1)}$$

The maximum angle involves the maximum height (1.124 + 0.0003″) and minimum sine bar length (3 – 0.0002″). I get 22.011°. The minimum angle involves the minimum height (1.124 – 0.0003″) and maximum sine bar length (3 + 0.0002″). Now I get 21.996°. In other words, my angle will be 22.004 ± 0.008°.

I welcome your comments and questions.

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