Avoiding Over Constraint, version 1.1

By R. G. Sparber

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Over constraint is one of those subjects that looks simple yet can be very hard to identify in a working machine.

A system that is under constrained can have unexpected movement. If it is fully constrained, all movement is predictable. If over constrained, unexpected movements can occur.

I will start with theory but provide real world examples along the way.

Here I have a dead flat surface (a plane), with a single point on it. I have marked this point with a cross hair and named it "O" for Origin. All measurements will be relative to this point.

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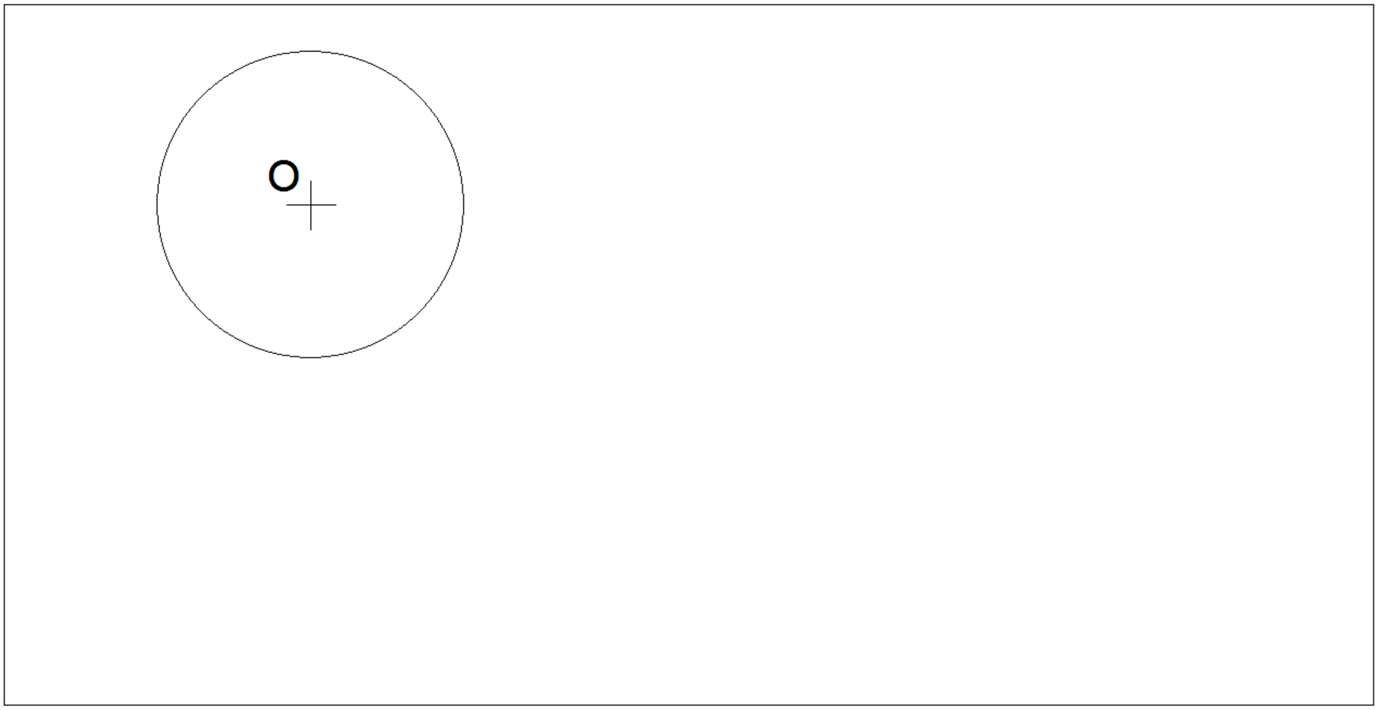
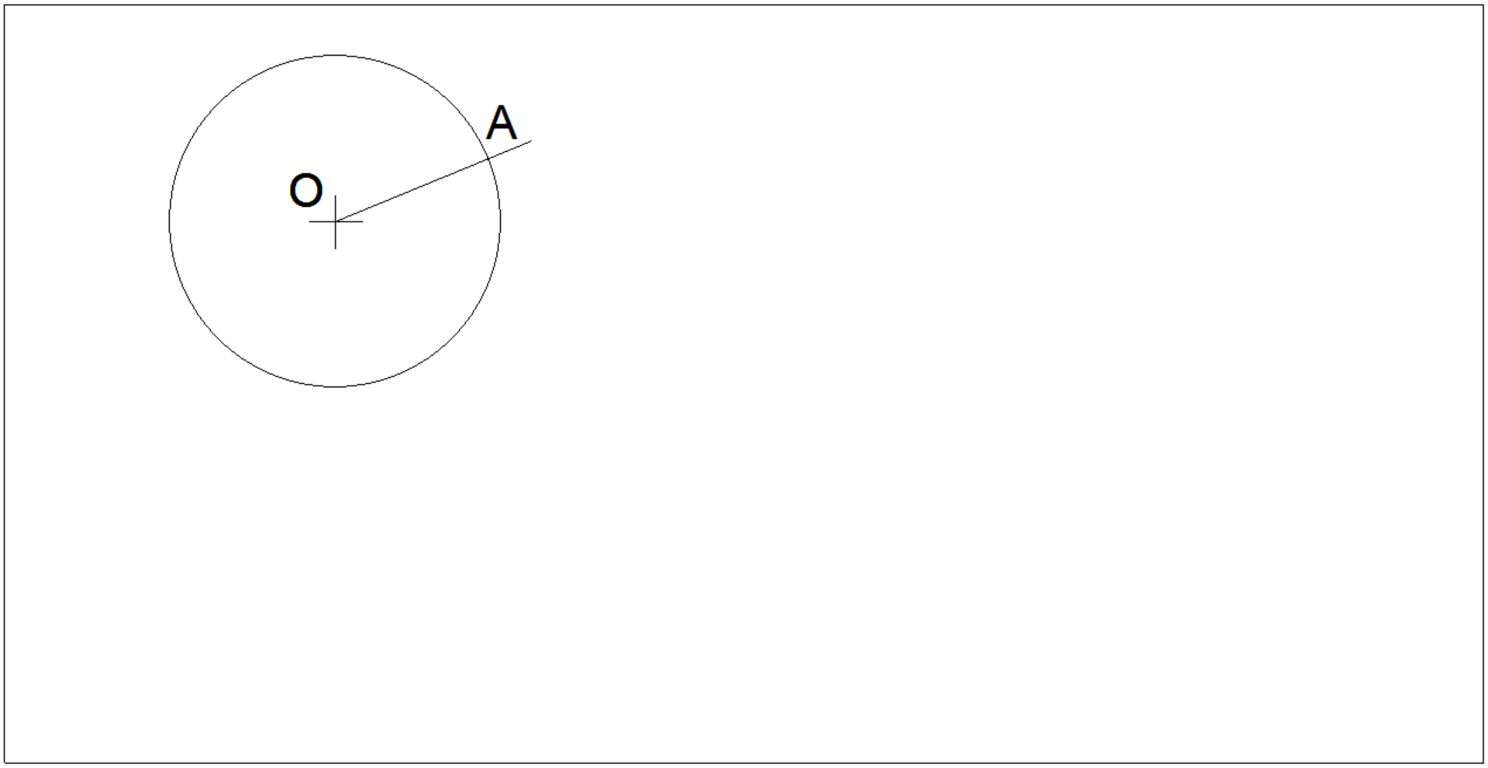
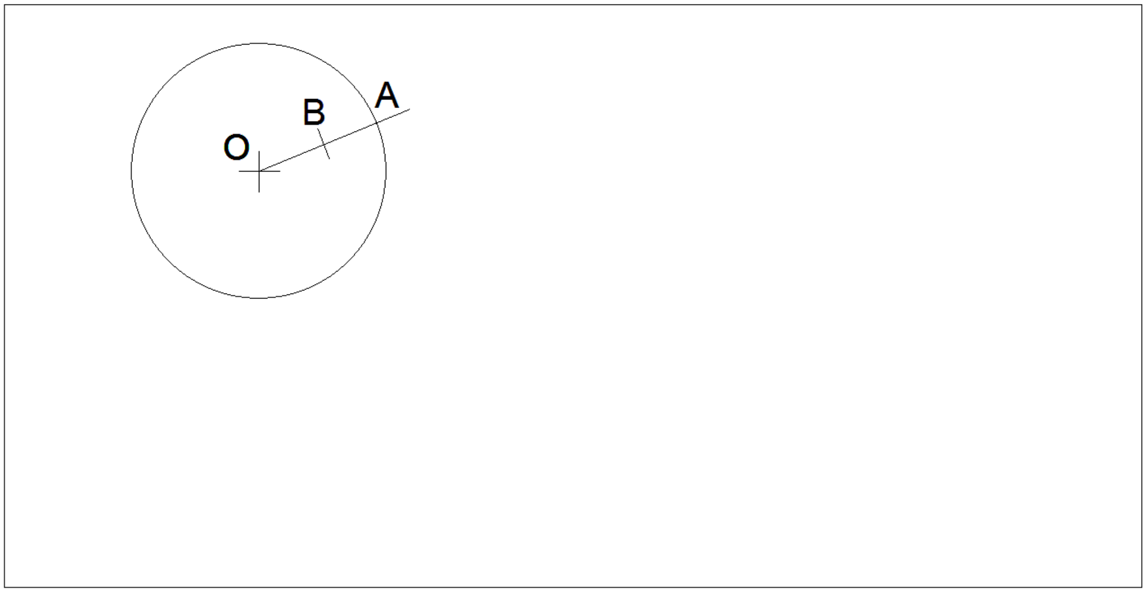


I can identify all points that are a given distance from my origin. This will define a circle. So here is my first constraint: only points a given distance from my origin are drawn. I still have an infinite number of points as long as the distance is non-zero. If the distance was zero, I'm back to having a single point.



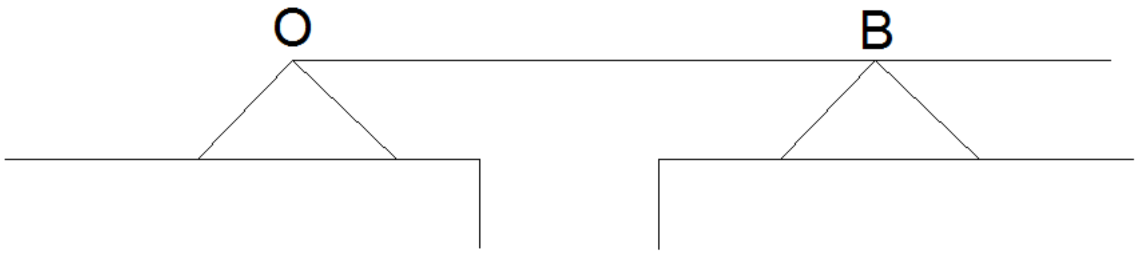
By drawing a straight line from the origin out beyond the circle, I can define a point, "A". This point is defined by the intersection of the circle and the line. I just went from an infinite number of points of equal distance from the origin down to one. I also defined a unique line. Call this line "OA".

Line OA must not be of zero length or we get back to having a single point.



Consider just the line. Sure O and A are on this line but so are an infinite number of other points. I could pick a new point on this line, call it "B". My OA line could also be called OB. As long as B is on the line and not at point O, the line is fully constrained. Yawn…

How about a practical application? Consider a bridge.



Point O anchors the bridge at one end. On the other shore I have point B. The bridge is a steel structure so will expand and contract as a function of temperature. Say the bridge is installed during the summer and is locked to its support at point B. As long as the temperature doesn't change, the length of the bridge between anchor points will exactly equal the distance between O and B.

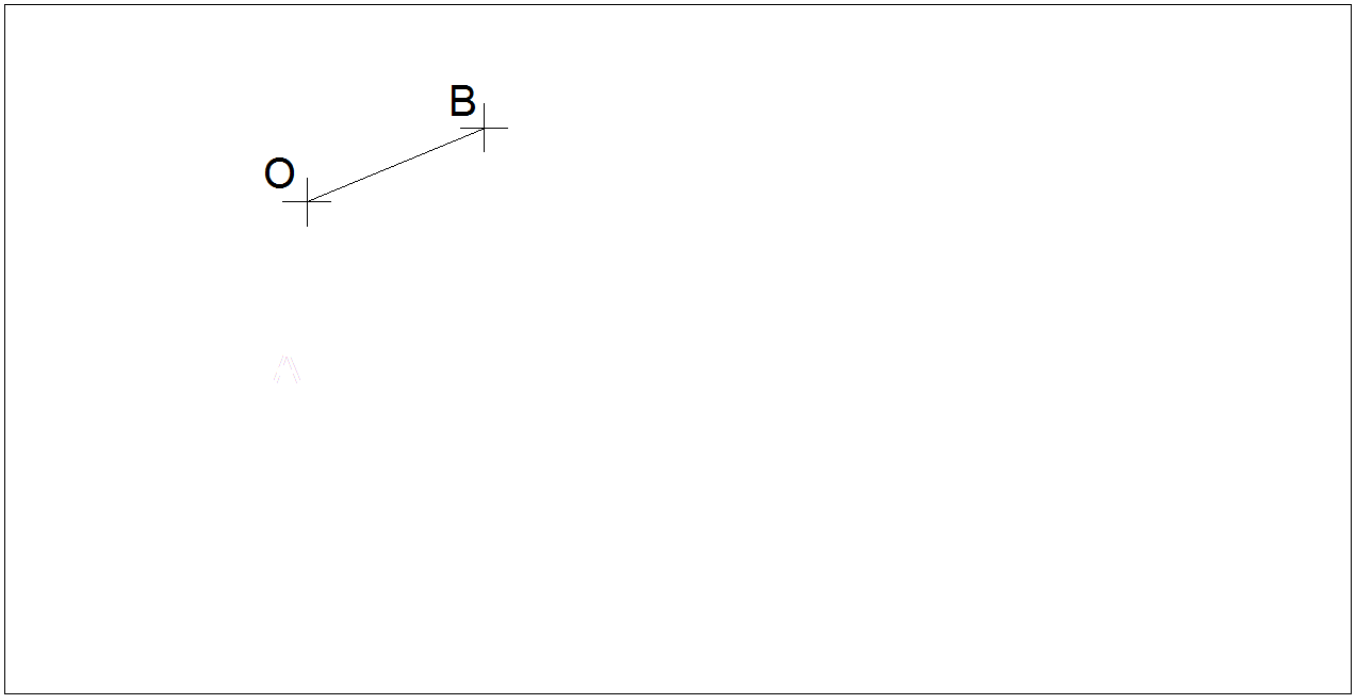
Now fast forward to the middle of winter. The steel has contracted due to the lower temperature. My anchor points do not move. This leaves us with a contradiction of sorts. The supports are one distance apart. The bridge is locked to these supports. But the bridge is shorter. So what gives? There is no way to tell. The weld at point O or B might break, the bridge might crack, or everything might bend a little and it all holds together. Not good.

The solution is rather elegant: do not secure the bridge at point B. Just let the bridge slide over point B. The alignment of the bridge is still defined by the line OB but variations in the length of the bridge are tolerated.

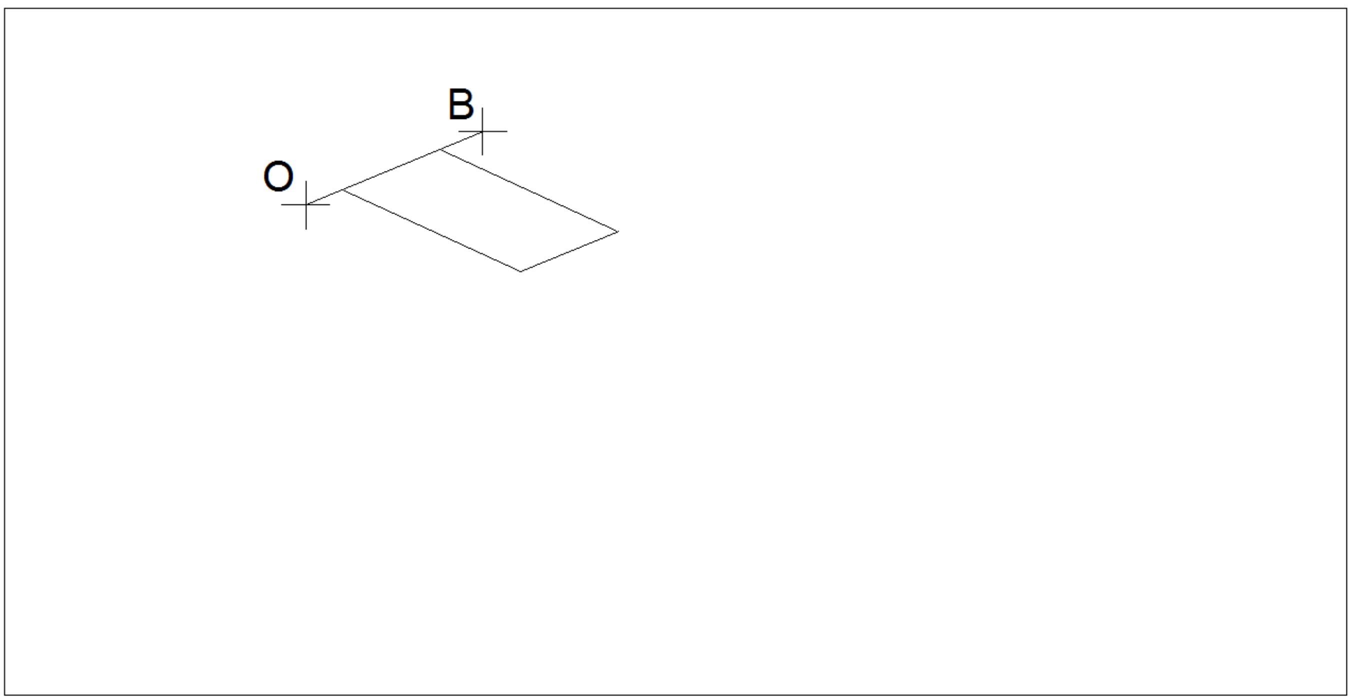
In fact, it is very common to see that both supports permit motion and the bridge is not secured at all. In this way, if either support sticks, the other one can still prevent an over constrained situation.

This same situation occurs when installing a scale for a Digital Read Out. If both ends of the scale are anchored down, what happens when things expand and contract? My scales are secured at one end and permitted to slide at the other.

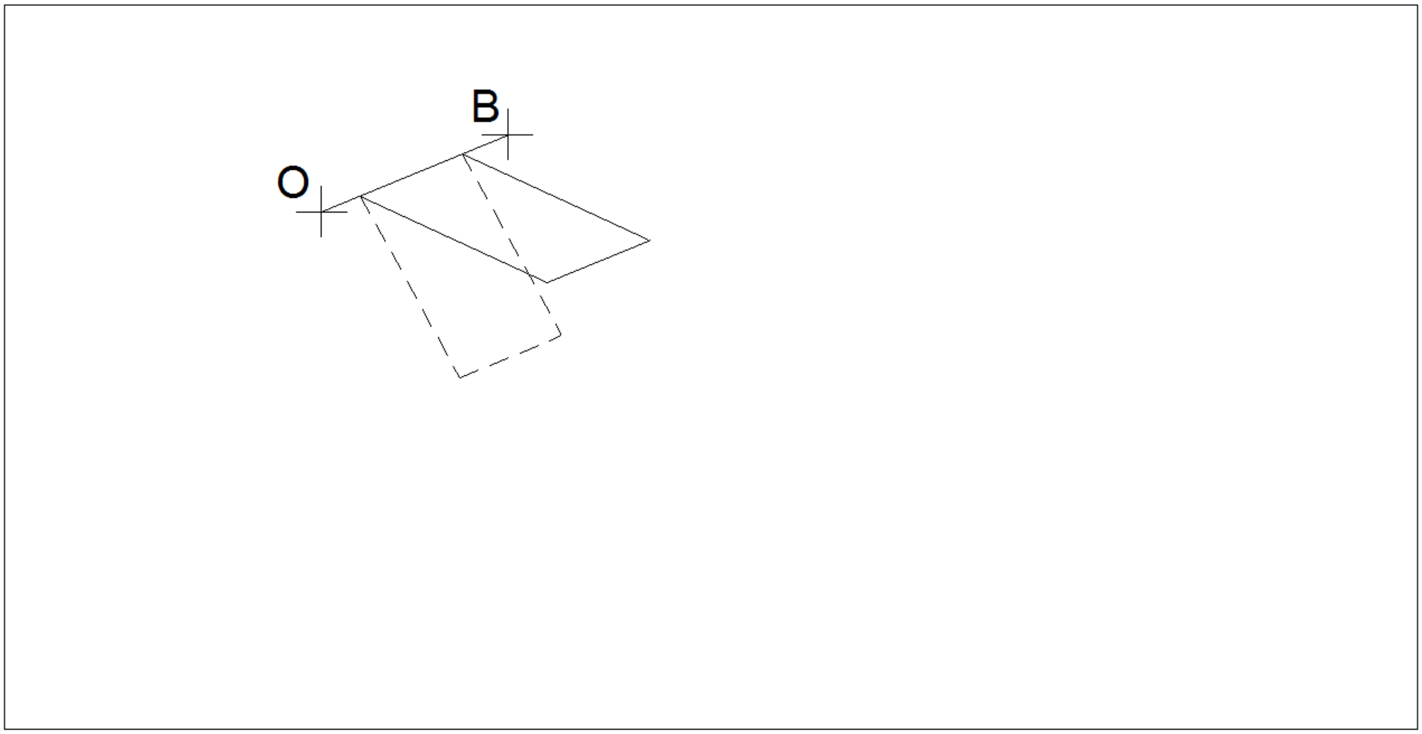
Back to the theory.



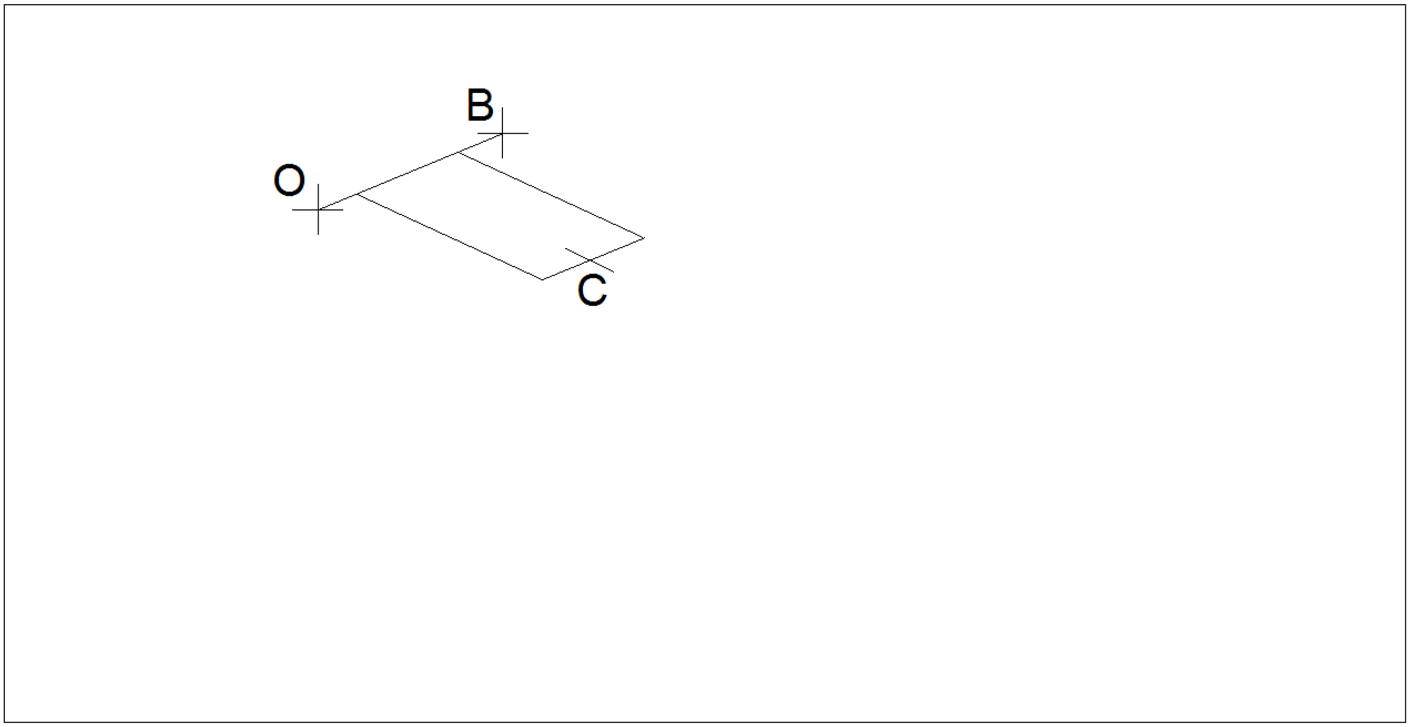
We are back to our non-zero length line defined by points O and B.



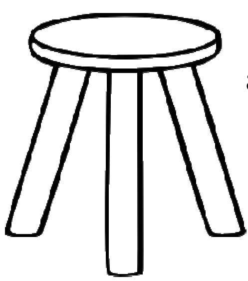
Think of line OB as the pivot pin in a hinge. The door is free to move as long as it pivots around this pin. I can define a plane that does the same thing. This plane contains the line OB and is free to move as long as OB stays on the plane.



Both my solid line plane and my new dashed line plane have line OB on their surface. Just as I have an infinite number of points on my circle shown on page 2, I have an infinite number of planes that contain the line OB. The plane is under constrained. That is a good thing if we are talking about a door which must open and close. It is a bad thing if the designer expects the plane not to move.



When I close the door and it latches, I have defined a third point on the plane. The theoretical model is shown here. Point "C" uniquely identifies one and only plane where before there were an infinite number of them. Nothing earth shaking about seeing that three points define a plane.

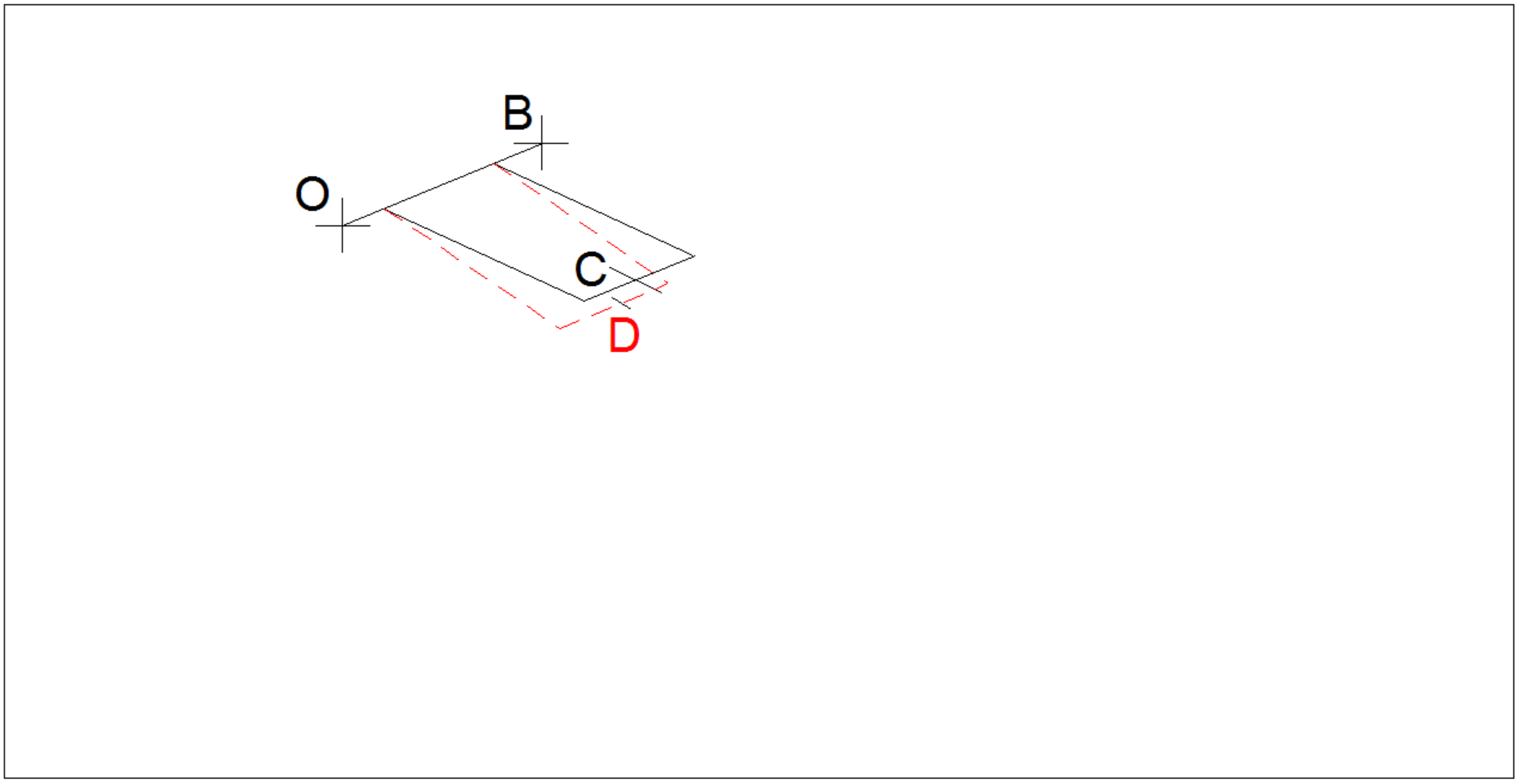


Anyone that has sat on a 3 legged stool knows it can't rock. Sit on a 4 legged stool, and it might not be steady. It doesn't matter if the legs are slightly different lengths or if the floor is not flat, 3 legs is always steady and 4 legs can rock. Three points define a plane.

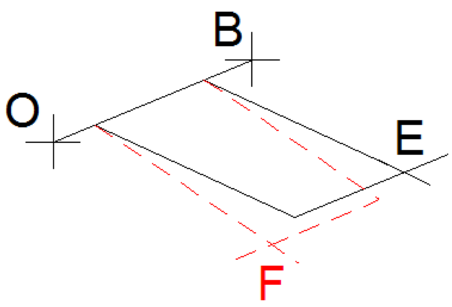
Notice that point C can be anywhere ***except*** on line OB. If point C was on line OB, then we just have a line and no plane has been defined.

Consider a 3 legged stool with all 3 legs forming a same straight line on the floor. It would just fall over as if it only had 2 legs.

Any new point that is on plane OBC could substitute for point C as long as it is not on line OB. Any new point that is not on plane OBC or line OB will define a new plane.



Even though the two planes share the same line, OB, having points C and D at different heights means there are two planes.

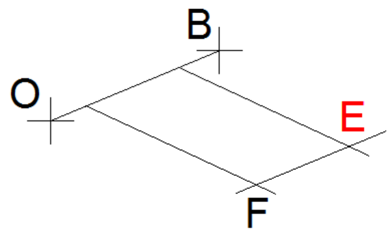


I have defined point E to be on the OBC plane. This means that plane OBE is identical to my old OBC plane. Similarly, I have defined point F to be on the OBD plane. Plane OBF is identical to my old OBD plane.

Say the back to legs of my stool are points O and B. The front right leg is point E and the front left leg is point F. The legs associated with points E and F are not exactly the same length although the floor is dead flat.

As I lean one way, my stool contacts the floor at points O, B, and E. then I shift my weight and contact the floor at points O, B, and F. Very annoying.

As you well know, there are only two choices. Either switch to a 3 legged stool or stuff something under the shorter leg.



If the stool stops rocking, you have effectively lowered point E until it is on plane OBF. The lack of rocking means you are sitting over a single plane rather than flipping between two planes.

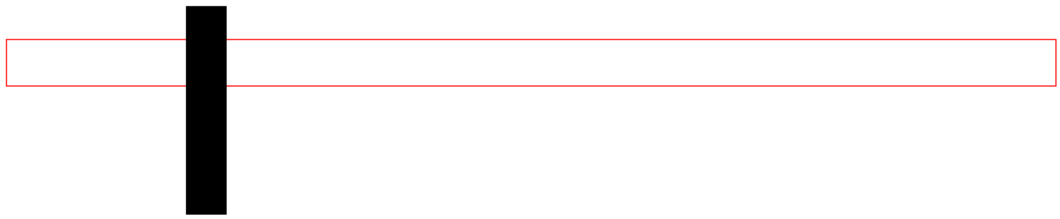
From a design standpoint, try to avoid defining a plane with 4 points. If you must, then consider the ability to adjust the height of one of these points.

Now, there are plenty of examples of one plane sliding on another plane. For example, a milling machine table sliding on its ways. Why doesn't that rock? Well… it does. However, the amount of rocking is extremely small ***or*** at any given time only 3 points are in contact ***or*** the table bends enough to conform to the ways.

Say your milling machine table rocked by ± 0.0001". The result would be a very slight ripple in the surface finish. Other factors, like the tram of the head could easily swamp this out.

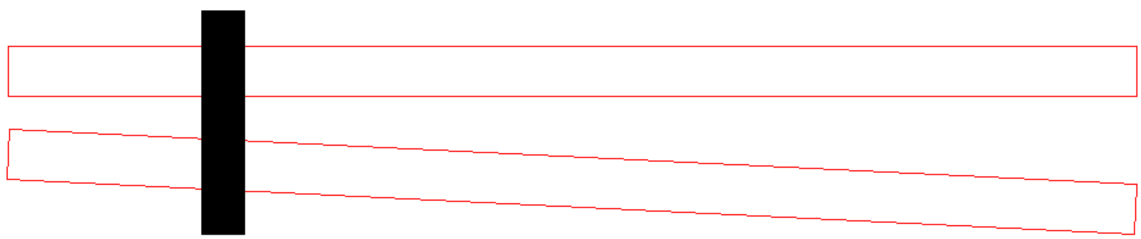
On the other hand, say the dovetail ways were not perfect planes. A variation of ± 0.0001" would mean that a gap of this amount would always have to exist or you would bind up as the table moved.

In other words, points not on the plane force machine tolerance. Failure to allow for these points means movements will bind up periodically.
Here is another example of a system prone to over constraint.



The red box is the outline of a precision rod. Assume it is perfectly straight and round. The black box is the outline of a block with a hole in it. The diameter of the hole in the block is equal to the diameter of the rod. OK, OK. The hole is a tiny bit larger so the block can move along the rod.

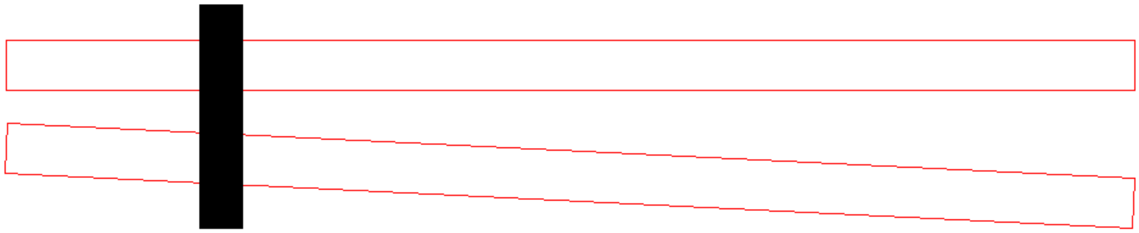
The rod defines a line identical to our OB line. The block defines a plane and is free to spin around the rod as shown on page 7. So we have an infinite number of planes here.



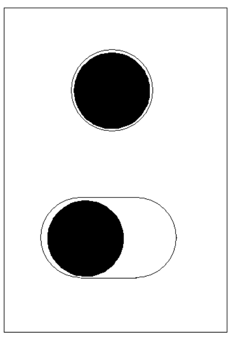
Say I bore a second hole in the block and fit a second rod. If this new hole is "much" larger than the rod, then the rods do not have to be parallel yet the block can move along the rods. The tighter the fit between new hole and its rod, the closer to parallel must be the rods.

It doesn't matter if this model is describing a simple block of metal with two holes drilled in it or a fancy linear bearing. The math is the same. The fit of the fancy linear bearing dictates how close to parallel must be the rods.

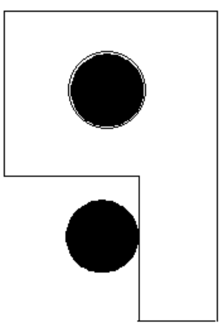
In the real world, the rods do have variation in diameter plus cannot be perfectly straight. These errors force the bearing designer to introduce some play between bearings and rod. Failure to do so means the bearings will bind up due to over constraint. This is why it is common to find the rods packaged with the bearings and sold as a system. It is also why they tend to cost a lot of money.



There are ways to side step this over constraint. The simplest way is to eliminate the second bar. The block will slide on the top bar and cannot bind up. That is not always sufficient.

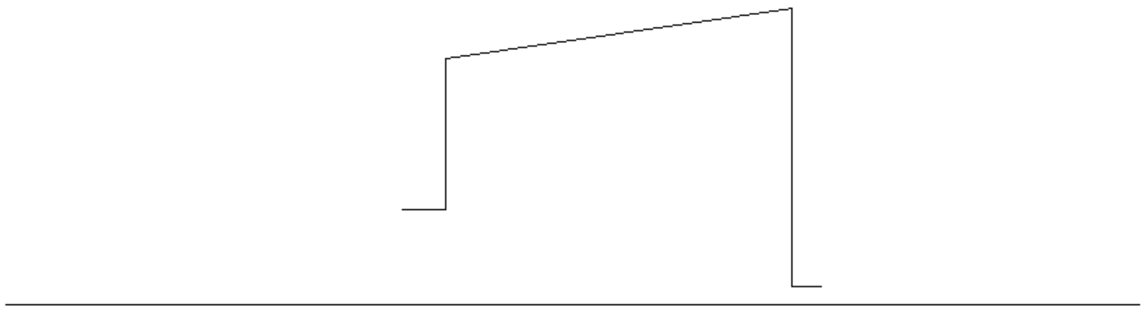


If the design requires that the block not rotate "too much" , it might be possible to have the second rod fit into a *slightly* elongated hole. This is a side view of the block. The wider the hole for the lower rod, the less stringent the alignment of the rods.

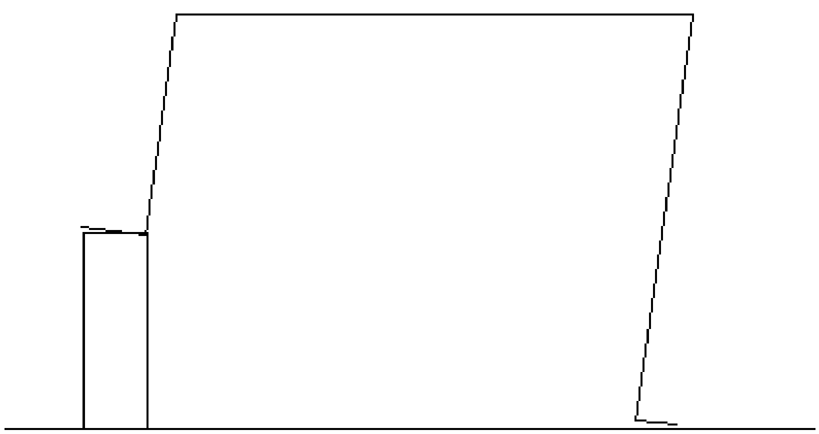


A variation on this idea is to replace the lower hole with a surface. As long as the slide block contacts the lower rod, the two rods do not have to be parallel.

Here is a problem I ran into recently. I have exaggerated the error to make it easier to see. Since only 2 dimensions are needed to appreciate the problem, I have avoided showing the real problem which is in 3 dimensions.

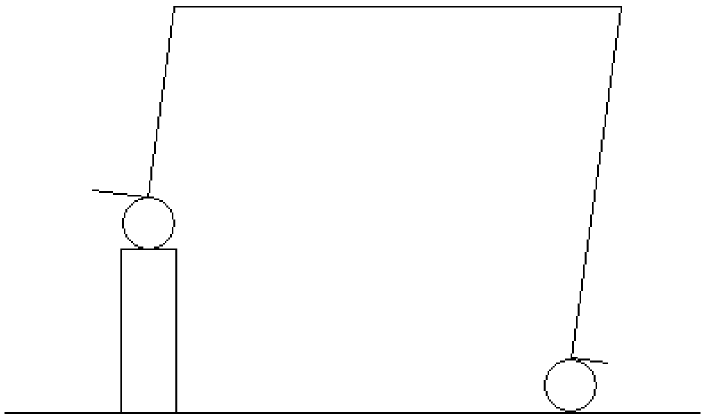


My reference surface is at the bottom. Above it is a plate supported by two legs. At the bottom of each leg is a flange that contains a hole. A bolt is run through the flange and into the reference surface to secure it. The plate must be parallel to the reference surface.

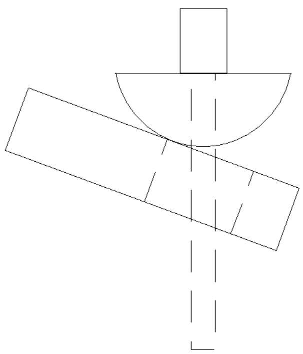


By using shims, I can jack up one leg and get the plate parallel to the reference surface. A single point is in contact between each flange and its supporting surface.

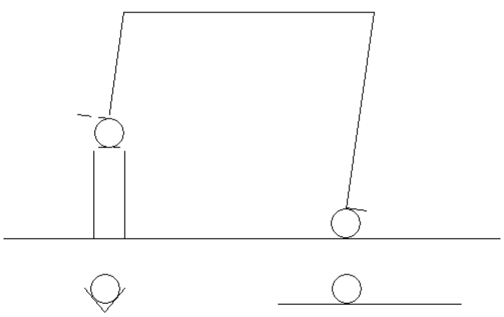
The problem becomes evident when I tighten the bolts. The flanges are not parallel to the reference surface. Tightening will bend metal. We are trying to force points not on the reference surface to be on this surface.



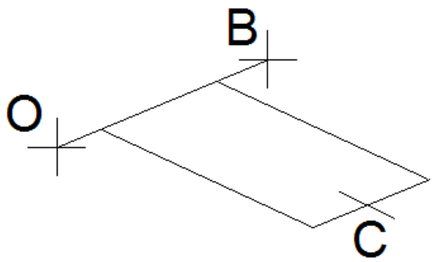
An elegant solution to this problem was given to me by John Herrmann. It involves placing a circle directly under each support leg. This insures a single point of contact. No distortion is possible because one and only one point of contact can exist between circle and reference surface.



A hold down bolt is then run through the flange, past the circle, and into the reference surface. The bolt feeds through a cylinder cut into a D cross section. The hole in the flange is oversized. In this way, the bolt can pull straight down into the reference surface yet apply pressure to the flange. The cut cylinder is rotated such that it sits flat on the flange.



In order to insure reproducible positioning, the circle on the left fits into a V. This permits the associated leg to pivot but not move away from this point on the reference surface. The circle on the right fits into a groove (sides not shown here). This permits this circle to move left and right but not in and out of the page.



In the full 3 dimensional situation, the circles are hardened ball bearings resting on hardened pads. The balls fit into threaded rods that enable me to change the length of each leg while the ball stays on its pad.

The first pad has a cone shaped hole in it. This permits the corresponding ball to rotate but not shift. That is the same as our O point.

The second pad has a V groove that permits the corresponding ball to move only left and right. That is the same as our B point.

The third ball is supported on a flat surface. It is free to move on the reference surface but not leave this surface. That is point C.

The result is a plane defined by the 3 balls that can be adjusted so the plate they support is parallel to the reference surface. The plate and associated legs are fully constrained but not over constrained.

Acknowledgement

Thanks to John Herrmann for helping me better see over constrained systems and providing the solution to my motor mount problem.

I welcome your comments and questions.

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