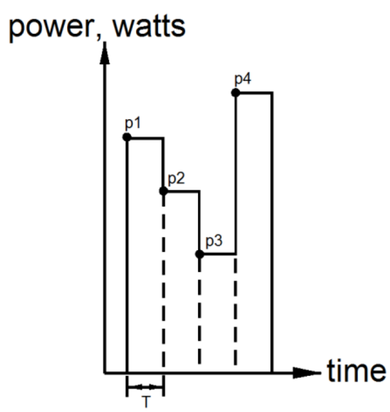
Arduino Software: How long before my battery is dead?, version 2.4

By R. G. Sparber

When powering an Arduino from a battery, it can be useful to be able to monitor available energy so you know when the battery will need charging. This requires measuring both battery voltage and current and doing calculations.

By multiplying battery voltage (in volts) times battery current (in amps), you calculate power (in watts).



Do this calculation every T milliseconds[2] and add them up: $(p1 \times T) + (p2 \times T) + (p3 \times T)$ ... You get a good approximation of watt-milliseconds which is a measure of power.

Divide this sum by 3.6×10^6 to convert from watt-milliseconds to watt-hours. The battery may be rated in watt-hours[3].

Batteries can also be rated in ampere-hours. You would do the same procedure except use current rather than power: $((I_1 \times T) + (I_2 \times T) + (I_3 \times T)$ ... and divide by 3.6×10^6.

If you then tell the software when the battery is fully charged, you can continuously subtract your running sum from this initial value to estimate remaining watt-hours or ampere-hours and therefore battery life.

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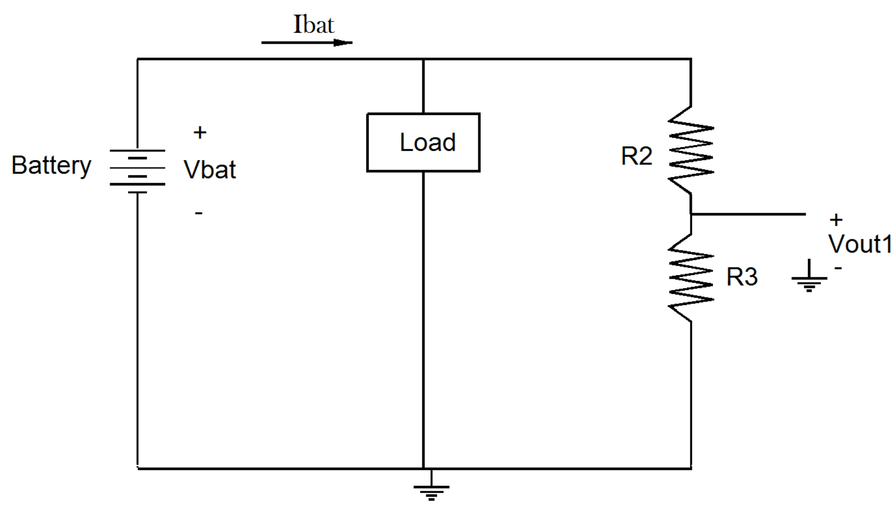
2 The faster power changes, the more often you must measure it.
3 See http://web.mit.edu/evt/summary_battery_specifications.pdf for how batteries are rated.



Measuring only battery voltage requires a two resistor voltage divider. The divider is needed because the battery voltage will be higher than the maximum input voltage of the Arduino.

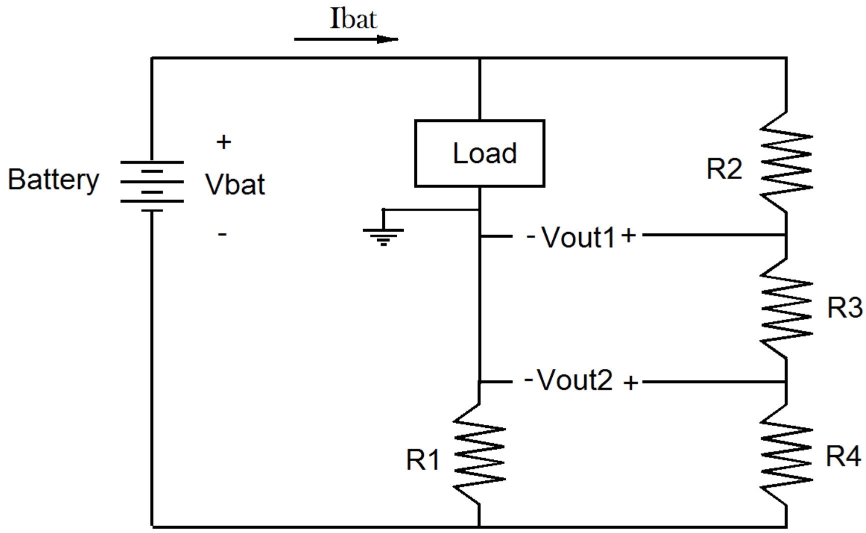
Measuring battery current can get complex. There are integrated circuits that measure current and output a voltage readable by the Arduino. But can it be done with the simplicity of the voltage divider?

Yes!

Contents

Measuring Voltage and Current[4]

This circuit will measure both battery voltage and battery current.



Battery Current

The Arduino software measures the voltage V_{out1} and V_{out2}. Then it calculates the ***battery current*** using these equations:

$$I_{bat} = k_1 V_{out1} - k_2 V_{out2} \qquad (1)$$

Where:

$$k_1 = \frac{R_4}{R_1 R_3} \qquad (2)$$

$$k_2 = \frac{1}{R_1}\left(\frac{R_4}{R_3} + 1\right) \qquad (3)$$

Given all resistors are in ohms, k_1 and k_2 are in $\frac{1}{ohms}$. With V_{out1} and V_{out2} in volts, I_{bat} will be in amps. See the Appendix I for how to measure R_1 which is typically less than one ohm. Appendix II shows an alternate way to measure k_1 and k_2.

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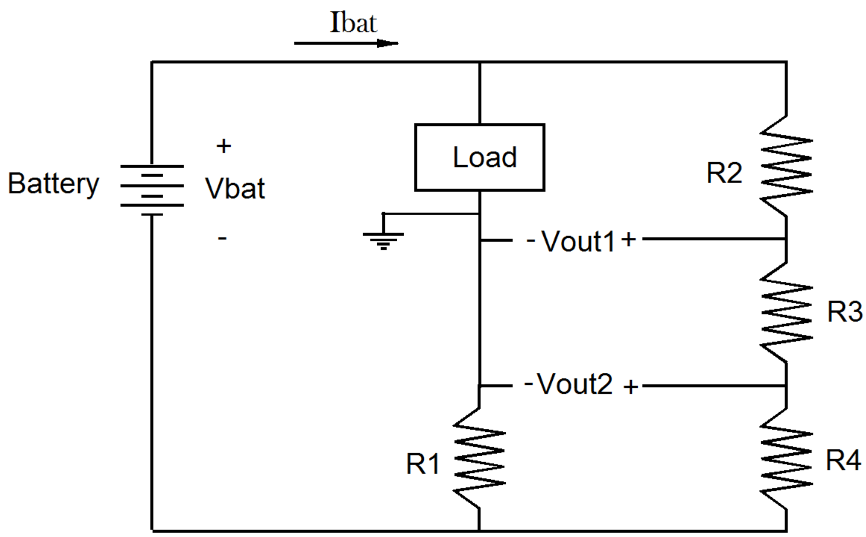
4 The down side of this circuit is you must be able to tolerate a supply voltage drop of about 0.5V at maximum output current. In my application we are using a LiPo battery with a minimum output voltage of 7.4V. The Arduino needs 5V. The low drop out 3 terminal regulator must therefore be able to work with an overhead of 7.4 - 0.5 - 5 = 1.9V. Not a problem. Plenty of Low Drop Out 5V regulators can handle that.

Battery Voltage



The Arduino software measures the voltage V_{out1} and V_{out2}. Then it calculates the ***battery voltage*** using these equations:

$$V_{bat} = k_3 (V_{out1} - V_{out2}) \qquad (4)$$

Where:

$$k_3 = \frac{R_2 + R_3 + R_4}{R_3} \qquad (5)$$

A more accurate way of calculating k_3 is to measure V_{bat} and the voltage between V_{out1} and V_{out2}. Then do a division:

$$k_3 = \frac{V_{bat}}{(V_{out1} - V_{out2})} \qquad (12)$$

Care must be taken to limit how fast V_{out1} and V_{out2} change relative to the sampling rate of the software. If necessary, place a low pass filter between each output voltage and the inputs of the Arduino. They prevent changes in current greater than the sampling rate of the software from being seen. Doing so would cause aliasing[5].

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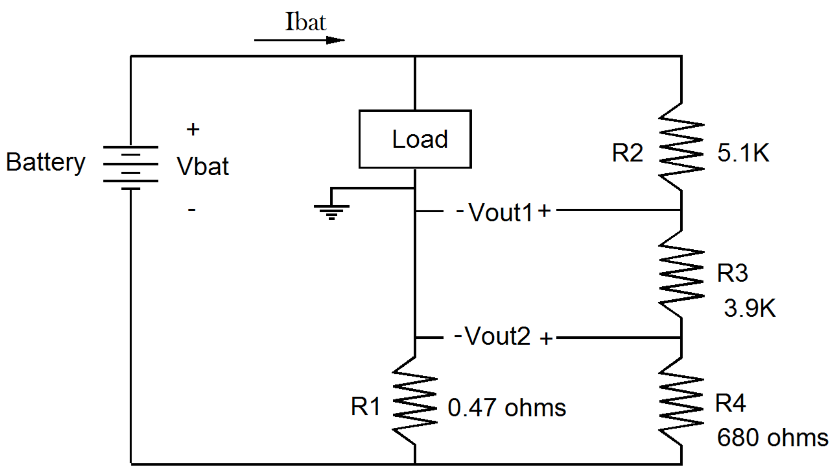
[5] https://en.wikipedia.org/wiki/Aliasing

Limiting Output Voltage

The Arduino can accept voltages at its inputs between zero and their supply voltage. We must select resistor values that prevent V_{out1} from rising above the supply voltage and prevent V_{out2} from falling below zero volts. These limitations have a major effect on selecting resistor values as will be seen in the Design Procedure section.

A Practical Example

Now let's consider some "reasonable" values to see what the software must do. On page 10 I we will address how to select these values.



This circuit monitors a battery with a maximum voltage of 10V, minimum voltage of 7V and a maximum output current of 1 amp. The two output voltages from the circuit vary between 0 and 5V so are compatible with the Arduino's analog inputs[6] assuming the device is powered from 5V.

$$\boldsymbol{I_{bat}} = k_1 V_{out1} - k_2 V_{out2} = \frac{\boldsymbol{V_{out1}}}{\boldsymbol{2.6956\ ohms}} - \frac{\boldsymbol{V_{out2}}}{\boldsymbol{0.4002\ ohms}} \qquad (1)$$

Where:

$$k_1 = \frac{R_4}{R_1 R_3} = \frac{680}{(0.47)(3900)} = \frac{1}{2.6956\ ohms} \qquad (2)$$

$$k_2 = \frac{1}{R_1}\left(\frac{R_4}{R_3} + 1\right) = \frac{1}{0.47}\left(\frac{680}{3900} + 1\right) = \frac{1}{0.4002\ ohms} \qquad (3)$$

and

$$\boldsymbol{V_{bat}} = k_3\,(V_{out1} - V_{out2}) = \boldsymbol{2.4821(V_{out1} - V_{out2})} \qquad (4)$$

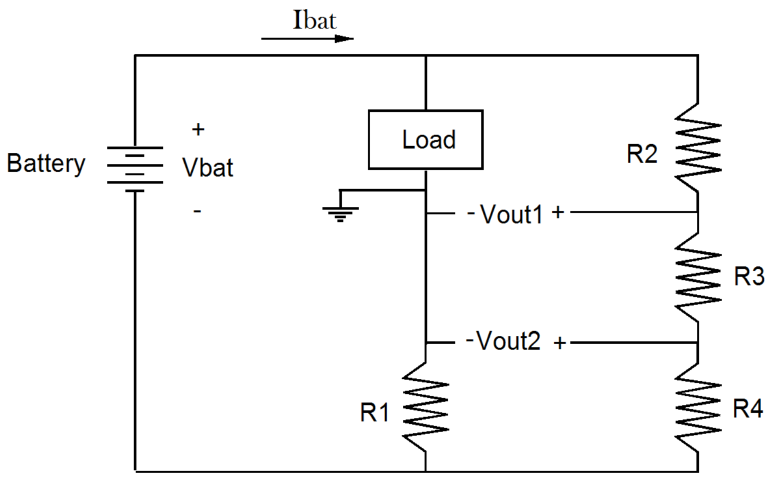
Where: $k_3 = \frac{R_2 + R_3 + R_4}{R_3} = \frac{5100+3900+680}{3900} = 2.4821 \qquad (5)$

[6] https://www.arduino.cc/en/Reference/AnalogRead

I am showing 4 places of significance to minimize round off error. Later I will present equations which address the accuracy of the circuit based on error in measuring V_{out1} and V_{out2} by the Arduino hardware.

R_1 is the shunt resistor. It can dissipate up to 0.5W so it would be prudent to use a 1W resistor. The remaining resistors can be 0.1W.

Circuit Operation Overview



The trick needed here is to notice that V_{out1} - V_{out2} is the voltage drop across R_3. This voltage difference divided by R_3 is the current flowing through R_3. This current also flows through R_2 and R_4 so we know their voltage drops which add up to the battery voltage:

$V_{bat} = \frac{V_{out1} - V_{out2}}{R_3} \times (R_2 + R_3 + R_4)$.

Knowing the current through R_3, we can calculate the voltage across R_4:

$$V_{R4} = \frac{V_{out1} - V_{out2}}{R_3} \times R_4$$

Then we can calculate the voltage across R_1:

$$V_{R1} = V_{out2} - V_{R4}.$$

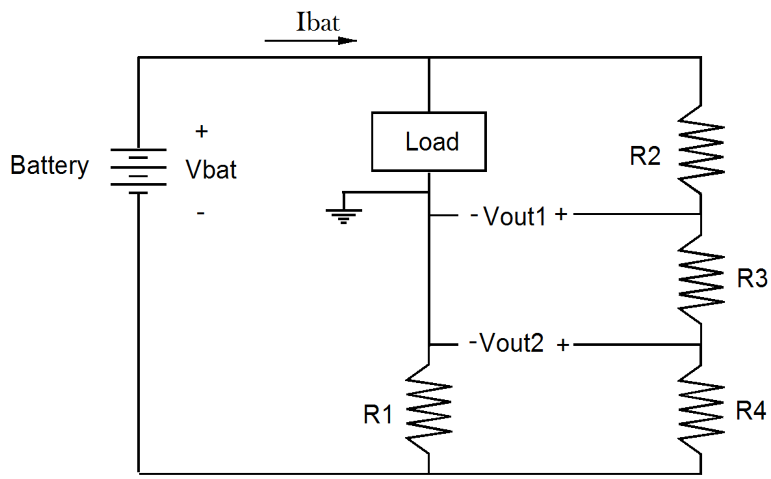
We know R_1 and that I_{bat} flows through the load, ground, and R_1. This tells us that

$$V_{R4} = R_1 \times I_{bat}$$

Using a bit of algebra, we can produce the equation that relates V_{out1} and V_{out2} to I_{bat}.

The remainder of this article presents the circuit analysis behind the equations.

Detailed Circuit Analysis



$$I_{R3} = \frac{V_{out1} - V_{out2}}{R_3} \qquad (6)$$

$$V_1 = I_{bat} R_1 \qquad (7)$$

$$V_{out2} = R_4 I_{R4} - I_{bat} R_1 \quad (8)$$

Solving (8) for I_{bat} we get

$$I_{bat} = \frac{R_4 I_{R4} - V_{out2}}{R_1} \qquad (9)$$

Note that $I_{R4} = I_{R3}$ as shown in (6). I can put (6) into (9) and write

$$I_{bat} = \frac{R_4 [\frac{V_{out1} - V_{out2}}{R_3}] - V_{out2}}{R_1} \qquad (10)$$

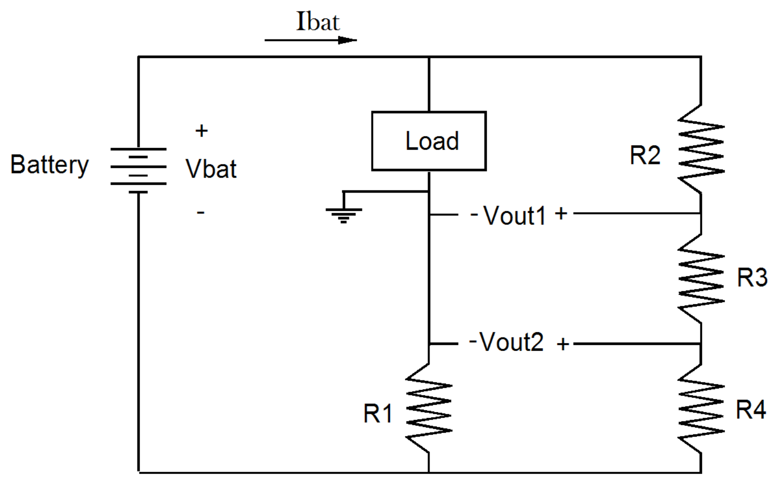
Which can be written as

$$I_{bat} = k_1 V_{out1} - k_2 V_{out2} \qquad (1)$$

Where:

$$k_1 = \frac{R_4}{R_1 R_3} \qquad (2)$$

$$k_2 = \frac{1}{R_1} (\frac{R_4}{R_3} + 1) \qquad (3)$$



Next, remember that V_{bat} equals the sum of the voltages across R_2, R_3, and R_4. The current flowing through R_2, R_3, and R_4 is V_{out1} minus V_{out2} divided by R_3. We can therefore say

$$V_{bat} = (R_2 + R_3 + R_4)(\frac{V_{out1} - V_{out2}}{R_3}) \quad (11)$$

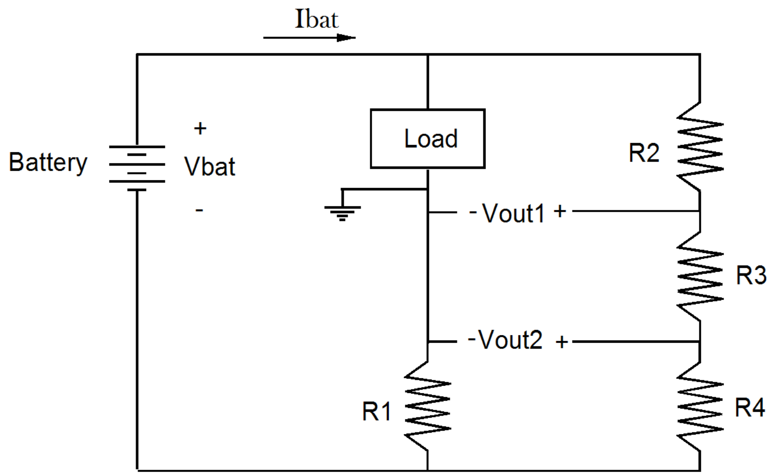
Which is

$$V_{bat} = k_3 \, (V_{out1} - V_{out2}) \quad (4)$$

Where:

$$k_3 = \frac{R_2 + R_3 + R_4}{R_3} \quad (5)$$

Design Procedure



I built an Excel spreadsheet that contains all design equations presented in this article. Once you understand the design process, the spreadsheet will make sense: http://rick.sparber.org/electronics/ASES.xlsx

Given values for the resistors, you have seen the equations that predict circuit behavior. But how do you find the resistor values?

We start by defining how the circuit must behave:

1. What is the maximum battery voltage? Call it V_{bb}.
2. What is the minimum battery voltage? Call it V_{mb}.
3. What is the maximum battery current? Call it I_{bb}.
4. What is the maximum safe voltage seen by the Arduino? Call it V_{cc}.
5. What is the maximum current flowing through R_3? Call it I_s.
6. What is the maximum voltage drop tolerable across R_1? Call it V_{sm}.

Furthermore, I will assume that the minimum battery current is zero and the minimum V_{out2} is zero.

$$\boldsymbol{R_2 = \frac{V_{bb}-V_{cc}}{I_s}} \qquad \text{(dp3)}$$

$$\boldsymbol{R_1 = \frac{V_{sm}}{I_{bb}}} \qquad \text{(dp4)}$$

$$\boldsymbol{R_4 = \frac{V_{sm}}{I_s \times \frac{V_{mb}}{V_{bb}}}} \qquad \text{(dp6)}$$

$$\boldsymbol{R_3} = \frac{\boldsymbol{V_{cc}}}{\boldsymbol{I_s}} - \boldsymbol{R_4} \qquad \text{(dp7)}$$

7. Select the closest standard values.
8. Calculate k_1, k_2, and k_3.
9. Verify V_{out1} and V_{out2} limits are not exceeded:

$$\boldsymbol{V_{cc}} =? \frac{\boldsymbol{R_3+R_4}}{\boldsymbol{R_2+R_3+R_4}} \times \boldsymbol{V_{bb}} \qquad \textbf{(dp8)}$$

$$\boldsymbol{0} =? \frac{\boldsymbol{R_4 \times V_{mb}}}{\boldsymbol{R_2+R_3+R_4}} - \boldsymbol{R_1} \times \boldsymbol{I_{bb}} \qquad \textbf{(dp9)}$$

If (dp8) shows that the calculated voltage is greater than Vcc, try rounding R_3 down to the next lowest value. This should also insure that the (dp9)'s calculated voltage is greater than zero.

Example:

Given V_{bb} = 10V, I_{bb} = 1 amp, V_{cc} = 5V, Is = 1 mA, V_{sm} = 0.47V

$$R_1 = \frac{V_{sm}}{I_{bb}} \qquad \text{(dp4)}$$

$$R_1 = \frac{0.47V}{1\ amp} = 0.47\ ohms$$

$$R_2 = \frac{V_{bb}-V_{cc}}{I_s} \qquad \text{(dp3)}$$

$$R_2 = \frac{10V-5V}{1\ mA} = 5K$$

$$R_4 = \frac{V_{sm}}{I_s} \qquad \text{(dp5)}$$

$$R_4 = \frac{0.47V}{1\ mA \times \frac{7}{10}} = 671\ ohms$$

$$R_3 = \frac{V_{cc}}{I_s} - R_4 \qquad \text{(dp6)}$$

$$R_3 = \frac{5V}{1\ mA} - 671\ ohms = 4.33K$$

Assuming 5% resistors, select R1 = 0.47 ohms, R2 = 5.1K, R3 = 4.7K, R4 = 680 ohms.

$$\boldsymbol{V_{cc} =?\ \frac{R_3+R_4}{R_2+R_3+R_4} \times V_{bb}} \qquad \textbf{(dp8)}$$

$= \frac{3.9+0.68}{5.1+4.7+0.68} \times 10 = 5.28V$ so above maximum input voltage.

$$\boldsymbol{0 =?\ \frac{R_4 \times V_{mb}}{R_2+R_3+R_4} - R_1 \times I_{bb}} \qquad \textbf{(dp9)}$$

$= \frac{0.68 \times 7}{5.1+4.7+0.68} - 0.47 \times 1\ amp = -0.072V$ so below the minimum input voltage.

We had taken the ideal value for R_3 of 4.33K and chosen the closest standard 5% value of 4.7K. But this caused the output voltages to be out of spec. So instead, try R_3 equal to 3.9K. Leave R_1 = 0.47 ohms, R_2 = 5.1K, and R_4 = 680 ohms.

$$\boldsymbol{V_{cc}} =? \frac{\boldsymbol{R_3+R_4}}{\boldsymbol{R_2+R_3+R_4}} \times \boldsymbol{V_{bb}} \qquad \textbf{(dp8)}$$

$$= \frac{3.9+0.68}{5.1+3.9+0.68} \times 10 = 4.73V$$ so now below maximum input voltage.

$$\boldsymbol{0} =? \frac{\boldsymbol{R_4 \times V_{mb}}}{\boldsymbol{R_2+R_3+R_4}} - \boldsymbol{R_1 \times I_{bb}} \qquad \textbf{(dp9)}$$

$$= \frac{0.68 \times 7}{5.1+3.9+0.68} - 0.47 \times 1\, amp = 0.0217V$$ so now above the minimum input voltage.

$$k_1 = \frac{R_4}{R_1 R_3} \qquad (2)$$

$$k_1 = \frac{680\ ohms}{0.47\ ohms \times 3.9K} = \frac{1}{2.6956\ ohms}$$

$$k_2 = \frac{1}{R_1} \left(\frac{R_4}{R_3} + 1\right) \qquad (3)$$

$$k_2 = \frac{1}{0.47\ ohms} \left(\frac{0.680K}{3.9K} + 1\right) = \frac{1}{0.4002\ ohms}$$

$$k_3 = \frac{R_2 + R_3 + R_4}{R_3} \qquad (5)$$

$$k_3 = \frac{5.1K + 3.9K + 0.680K}{5.1K} = 1.898$$

Going back to our current equation:

$$I_{bat} = k_1 V_{out1} - k_2 V_{out2} \qquad (1)$$

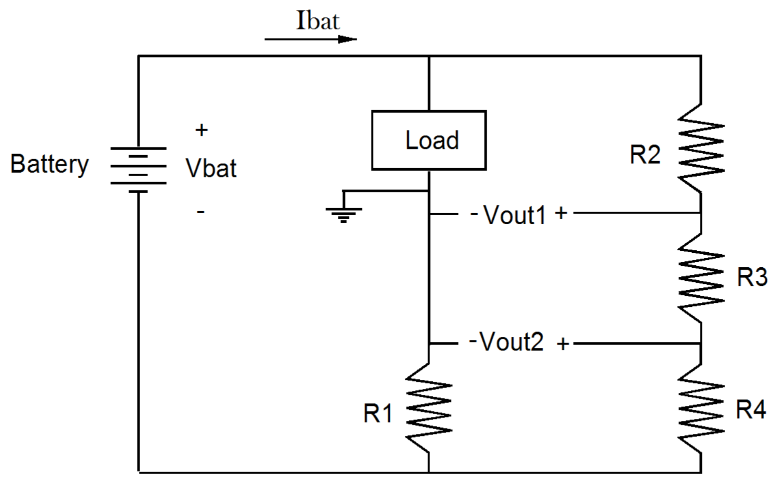
$$\boldsymbol{I_{bat}} = \frac{\boldsymbol{V_{out1}}}{\boldsymbol{2.6956\ ohms}} - \frac{\boldsymbol{V_{out2}}}{\boldsymbol{0.4002\ ohms}}$$

Going back to our voltage equation:

$$V_{bat} = k_3 \left(V_{out1} - V_{out2}\right) \qquad (4)$$

$$\boldsymbol{V_{bat} = 1.898 \ \left(V_{out1} - V_{out2}\right)}$$

Design Equations Derivation



We start by defining how the circuit must behave:

1. What is the maximum battery voltage? Call it V_{bb}.
2. What is the minimum battery voltage? Call it V_{mb}.
3. What is the maximum battery current? Call it I_{bb}.
4. What is the maximum voltage seen by the Arduino? Call it V_{cc}.
5. What is the maximum current flowing through R_3? Call it I_s.
6. What is the maximum voltage drop tolerable across R_1? Call it V_{sm}.

Furthermore, I will assume that the minimum battery current is zero and the minimum V_{out2} is zero.

Then we make a few observations:

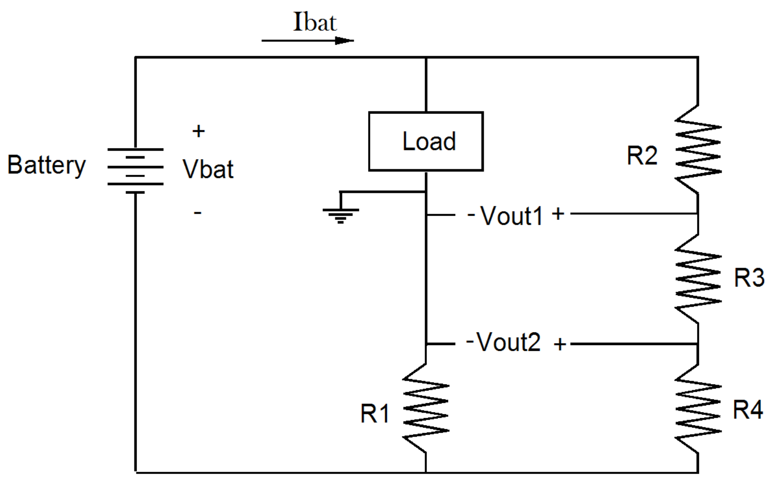
I_s exists when the battery is at V_{bb}.

$$\frac{V_{bb}}{I_s} = R_2 + R_3 + R_4 \quad \text{(dp1)}$$

V_{out1} will always be larger than V_{out2} and will be at its maximum value when the battery is at V_{bb}. Furthermore, the voltage drop across R_1 will be at its smallest when I_{bat} is at zero. This will cause the bottom of R_4 to be at zero volts (virtual ground) and will contribute to placing V_{out1} at its maximum value.

$$\frac{V_{cc}}{I_s} = R_3 + R_4 \qquad \text{(dp2)}$$

Substituting (dp2) into (dp3)



$$\frac{V_{bb}}{I_s} = R_2 + \frac{V_{cc}}{I_s}$$

$$R_2 = \frac{V_{bb-}V_{cc}}{I_s} \qquad \text{(dp3)}$$

The maximum voltage drop across R_1 is V_{sm} and this will occur at the maximum battery current of I_{bb}:

$$R_1 = \frac{V_{sm}}{I_{bb}} \qquad \text{(dp4)}$$

At maximum battery current, V_{out2} will, by design, be at zero volts. However, the voltage drop across R4 will be at its smallest when V_{bat} is at its minimum value, V_{mb}. This means that the current through R_4, I_{smb}, will be

$$I_{smb} = \frac{V_{mb}}{R_2+R_3+R_4}$$

and the resulting voltage drop across R_4 will be $R_4 \times I_{smb}$

$$V_{out2} = -V_{R1} + V_{R4}$$

$$0 = -(R_1 \times I_{bb}) + (R_4 \times \frac{V_{mb}}{R_2 + R_3 + R_4})$$

$$(R_1 \times I_{bb}) = (R_4 \times \frac{V_{mb}}{R_2+R_3+R_4}) \qquad \text{(dp5)}$$

Looking back at (dp2)

$$\frac{V_{cc}}{I_s} = R_3 + R_4 \qquad \text{(dp2)}$$

$$\boldsymbol{R_2 = \frac{V_{bb} - V_{cc}}{I_s}} \qquad \text{(dp3)}$$

$$\boldsymbol{R_1 = \frac{V_{sm}}{I_{bb}}} \qquad \text{(dp4)}$$

Put (dp2), (dp3), and (dp4) into (dp5)

$$\left(\frac{V_{sm}}{I_{bb}} \times I_{bb}\right) = \left(R_4 \times \frac{V_{mb}}{\frac{V_{bb} - V_{cc}}{I_s} + \frac{V_{cc}}{I_s}}\right)$$

$$V_{sm} = \left(R_4 \times I_s \times \frac{V_{mb}}{V_{bb} - V_{cc} + V_{cc}}\right)$$

$$V_{sm} = \left(R_4 \times I_s \times \frac{V_{mb}}{V_{bb}}\right)$$

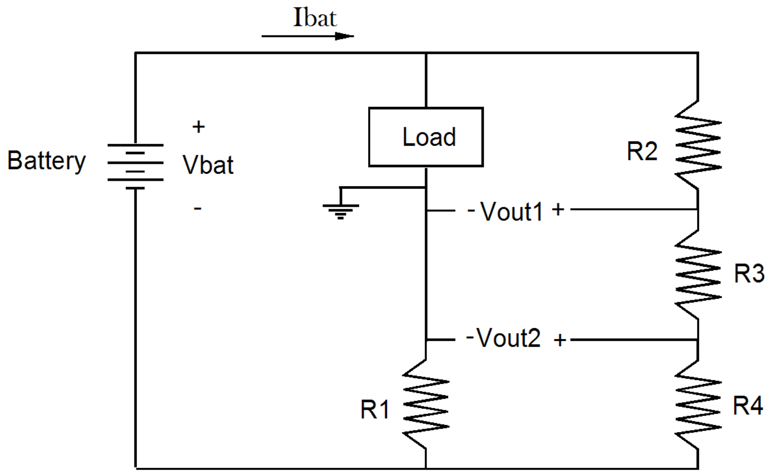
$$\boldsymbol{R_4 = \frac{V_{sm}}{I_s \times \frac{V_{mb}}{V_{bb}}}} \qquad \text{(dp6)}$$

Putting (dp6) into (dp2) gives us

$$\boldsymbol{R_3 = \frac{V_{cc}}{I_s} - R_4} \qquad \text{(dp7)}$$

We can now use (dp4) to find R_1, (dp3) to find R_2, (dp6) to find R_4, and (dp7) to find R_3.

1. the maximum battery voltage is V_{bb}.
2. the minimum battery voltage is V_{mb}.
3. maximum battery current is I_{bb}.
4. the maximum voltage seen by the Arduino is V_{cc}.
5. the maximum current flowing through R_3 is I_s.
6. the maximum voltage drop tolerable across R_1 is V_{sm}.



We can verify these resistor values are correct by calculating the maximum V_{out1} and the minimum V_{out2} for each of their worst cases.

For V_{out1} maximum (i.e. V_{cc}), it will occur when V_{bat} is at maximum (i.e. V_{bb}) and the voltage across R_1 is at minimum (i.e. when I_{bat} is zero).

$$V_{out1\,max} = V_{cc} =? \frac{R_3+R_4}{R_2+R_3+R_4} \times V_{bb} \qquad \text{(dp8)}$$

For V_{out1} minimum (i.e. 0), it will occur when V_{bat} is at minimum (i.e. V_{mb}) and the voltage across R1 is at maximum (i.e. R_1 X I_{bb}).

$$V_{out2\,min} = 0 =? \frac{R_4 \times V_{mb}}{R_2+R_3+R_4} - R_1 \times I_{bb} \qquad \text{(dp9)}$$

The final step is to test out all of the equations. I will assume:

V_{bb} = 10, V_{mb} = 7, Ibb = 1 amp, V_{cc} = 5, I_s = 1 mA, V_{sh} = 0.47

Then

$$\boldsymbol{R_2} = \frac{\boldsymbol{V_{bb} - V_{cc}}}{\boldsymbol{I_s}} \qquad \text{(dp3)}$$

$$R_2 = \frac{10-5}{1\ mA} = 5K$$

$$\boldsymbol{R_1} = \frac{\boldsymbol{V_{sm}}}{\boldsymbol{I_{bb}}} \qquad \text{(dp4)}$$

$$R_1 = \frac{0.47}{1\ amp} = 0.47\ ohms$$

$$\boldsymbol{R_4} = \frac{\boldsymbol{V_{sm}}}{\boldsymbol{I_s} \times \frac{\boldsymbol{V_{mb}}}{\boldsymbol{V_{bb}}}} \qquad \text{(dp6)}$$

$$R_4 = \frac{0.47}{1\ mA \times \frac{7}{10}} = 0.671K$$

$$\boldsymbol{R_3} = \frac{\boldsymbol{V_{cc}}}{\boldsymbol{I_s}} - \boldsymbol{R_4} \qquad \text{(dp7)}$$

$$R_3 = \frac{5}{1\ mA} - 671\ ohms = 4.33K$$

$$V_{out1\ max} = V_{cc} =? \frac{R_3 + R_4}{R_2 + R_3 + R_4} \times V_{bb} \qquad \text{(dp8)}$$

$V_{out1\ max} = 5 =? \frac{4.33K + 0.671K}{5K + 4.33K + 0.671K} \times 10$ = 5 so V_{out1} maximum is correct

$$V_{out2\ min} = 0 =? \frac{R_4 \times V_{mb}}{R_2 + R_3 + R_4} - R_1 \times I_{bb} \qquad \text{(dp9)}$$

$V_{out2\ min} = 0 =? \frac{0.671K \times 7}{5K + 4.33K + 0.671K} - 0.47\ ohms \times 1\ amp = 0.000$ so V_{out2} minimum is correct.

We must verify the Arduino is still seeing voltages within specifications.

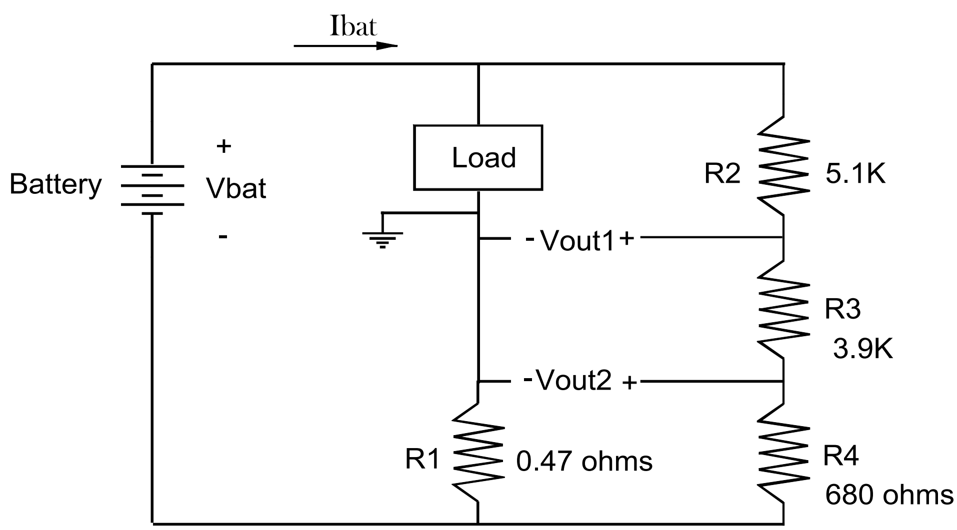
Name	Ideal value	Closest value
R1	0.47 ohms	0.47 ohms
R2	5K	5.1K
R3	4.33K	4.7K
R4	671 ohms	680 ohms

When these values are plugged into (dp8) and (dp9) I discovered that the maximum V_{out1} was 5.28V and the minimum V_{out2} was -0.072V. Both are out of spec.

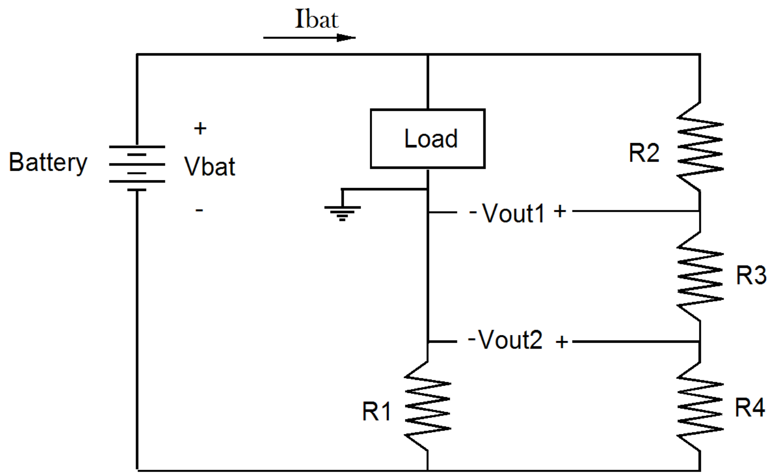
Notice that I rounded R3 up. This caused V_{out1} to rise and caused V_{out2} to fall. What if I round R_3 down?

Name	Ideal value	Closest value
R1	0.47 ohms	0.47 ohms
R2	5K	5.1K
R3	4.33K	3.9K
R4	671 ohms	680 ohms

Then (dp8) and (dp9) tell me that V_{out1} maximum is 4.73V and V_{out2} minimum is +0.0217V. Both of these values are safe for an Arduino powered from 5V.



Estimating Circuit Accuracy



Accuracy has two components. The first relates to precisely determining k_1, k_2, and k_3. The second component relates to measuring V_{out1} and V_{out2}.

We could precisely measure each resistor and then calculate k_1, k_2, and k_3.But for k_3, it is possible to be more accurate by measuring two voltages in order to directly calculate the constant.

Consider (4)

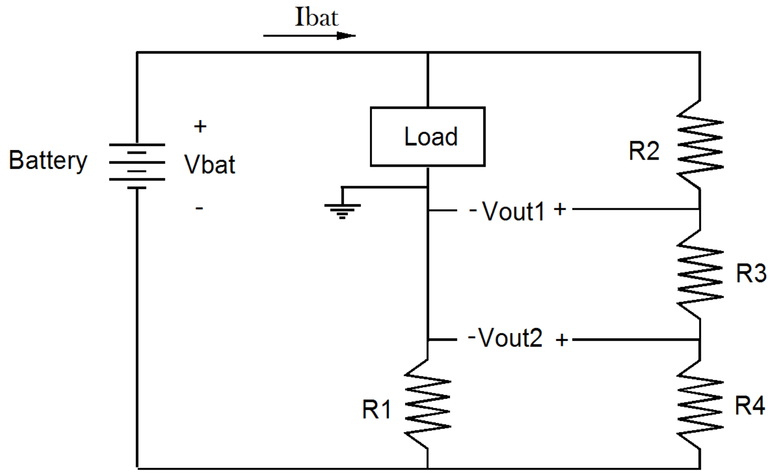
$$V_{bat} = k_3 (V_{out1} - V_{out2}) \qquad (4)$$

Rearranging terms we get

$$k_3 = \frac{V_{bat}}{(V_{out1} - V_{out2})} \qquad (11)$$

Notice here that V_{out1} - V_{out2} is the voltage across R_3.

$$k_3 = \frac{V_{bat}}{V_{R3}} \qquad (12)$$



$$k_3 = \frac{V_{bat}}{V_{R3}} \qquad (12)$$

(12) tells us to measure the battery voltage and the voltage across R_3. Divide and we get k_3. If both readings were taken with the same meter set to the same scale, meter error would tend to cancel giving us the most accurate measure of k_3.

$$V_{bat} = k_3\,(V_{out1} - V_{out2}) \qquad (4)$$

In order to determine error sensitivity, we take the derivative of V_{bat} with respect to voltage:

$$dV_{bat} = k_3 d\,V_{out1} - k_3 d\,V_{out2} \qquad (13)$$

Where

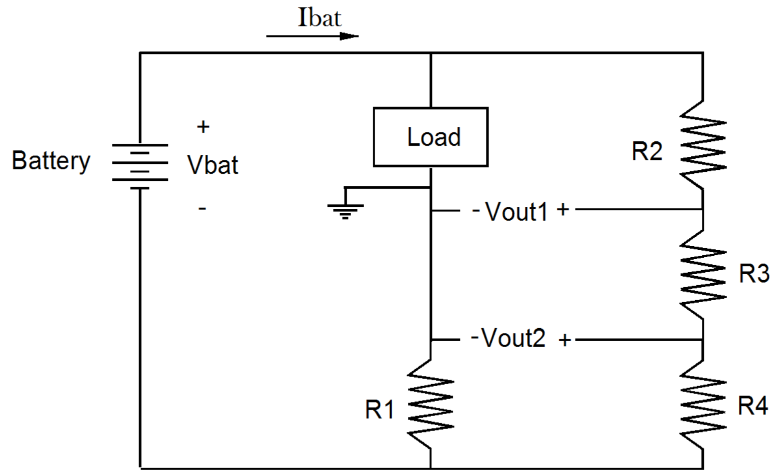
dV_{bat} is the change in calculated battery voltage
dV_{out1} is the error in voltage associated with V_{out1}
dV_{out2} is the error in voltage associated with V_{out2}

For example, given a k_3 of 2, for a 5 mV error in reading V_{out1} and V_{out2} we get

$$dV_{bat} = (2)(5\,mV) - (2)(5\,mV) = 0$$

This is telling us that if the error in V_{out1} and V_{out2} move in the same direction and by the same amount, the result is zero error.

Next we look at the current measuring part of the circuit



$$I_{bat} = k_1 V_{out1} - k_2 V_{out2} \qquad (1)$$

Where:

$$k_1 = \frac{R_4}{R_1 R_3} \qquad (2)$$

$$k_2 = \frac{1}{R_1} \left(\frac{R_4}{R_3} + 1\right) \qquad (3)$$

I take the derivative of (1) and get

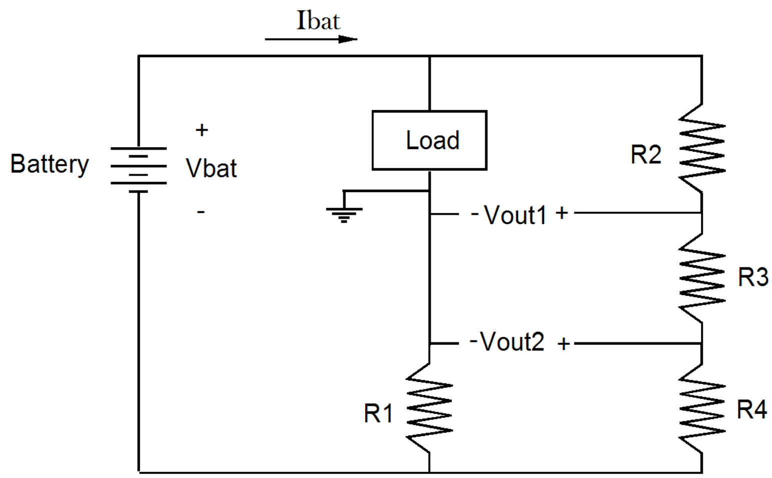
$$dI_{bat} = k_1\, dV_{out1} - k_2\, dV_{out2} \qquad (15)$$

Where

dI_{bat} is the change in calculated current
dV_{out1} is the error in volts associated with V_{out1}
dV_{out2} is the error in volts associated with V_{out2}

Since k_1 and k_2 are not equal, errors in measuring V_{out1} and V_{out2} do not tend to cancel.

Plugging in the presented values for k1 and k2, we get



$$dI_{bat} = \frac{dV_{out1}}{3.9\ ohms} - \frac{dV_{out2}}{0.5266\ ohms}$$

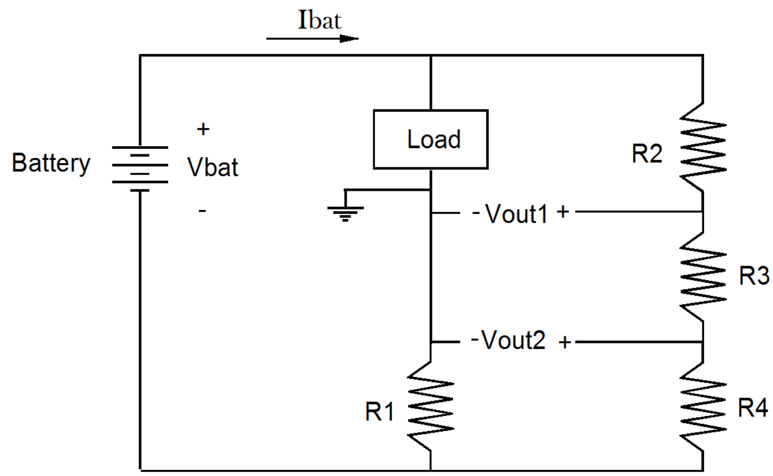
Say the error for Vout1 and Vout2 were again 5 mV.

$$dI_{bat} = \frac{5\ mV}{3.9\ ohms} - \frac{5\ mV}{0.5266\ ohms}$$ = -0.00821 amps = -8.21 mA

This tells us that if V_{out1} and V_{out2} both contained a 5 mV error, I_{bat} would be off by 8 mA. When measuring near 1 amp, this error is probably acceptable but when measuring at 8 mA, the error would be 100%. What saves us here is that a small current means a small energy usage so the error has little effect on our energy calculation.

Prototype Testing

I built the circuit to see if it really works as predicted.



Using a Fluke77 meter, I measured resistors

R3 = 3.97K
R4 = 464 ohms

I then used a 70 ohm 5 watt resistor and a 10V supply to measure R_1. My meter read 138 mA flowing through R_1 and 502 mV across it. This means $R_1 = \frac{138\ mV}{275\ mA} = 0.502\ ohms$

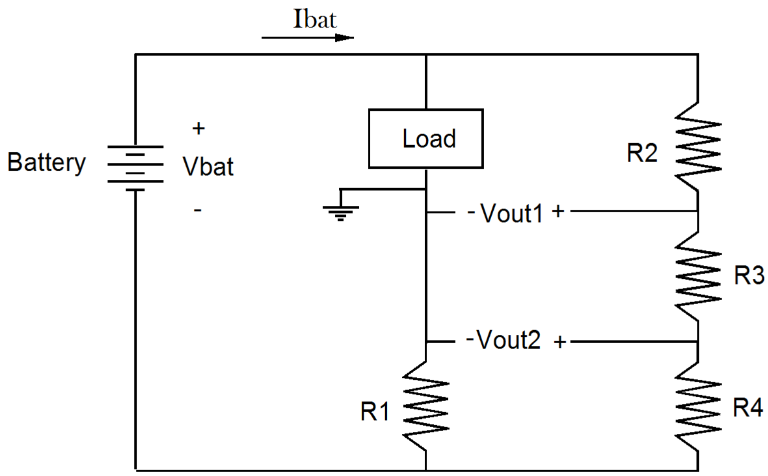
$$k_1 = \frac{R_4}{R_1 R_3} = 0.23282 \frac{1}{ohms} \quad (2)$$

$$k_2 = \frac{1}{R_1}\left(\frac{R_4}{R_3} + 1\right) = 2.22485 \frac{1}{ohms} \quad (3)$$

I then built the circuit and measured the voltage across the battery and R_3 to get the last constant.

$$k_3 = \frac{V_{bat}}{V_{R3}} = 2.105 \quad (12)$$

Note that I didn't bother to measure R_2 other than to verify it was marked correctly. In finding k_3, I measure the effects of R_2.



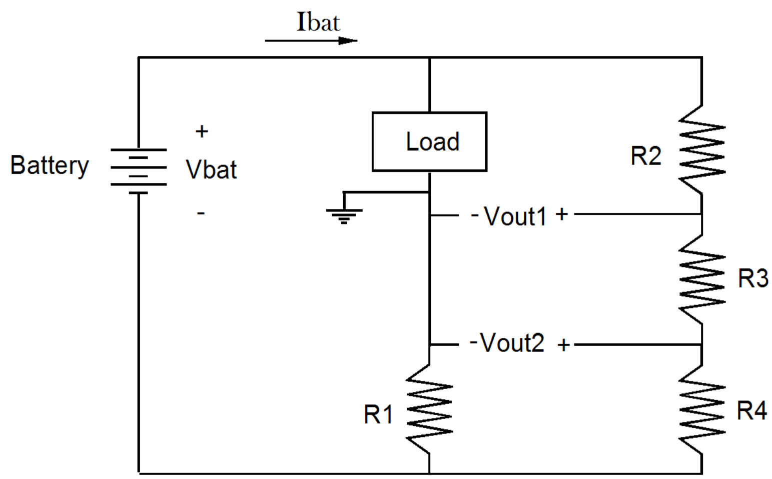
$k_3 = \frac{V_{bat}}{V_{R3}} = \frac{R_2 + R_3 + R_4}{R_3}$ (12) and (5)

$I_{bat} = k_1 V_{out1} - k_2 V_{out2}$ (1)

$I_{bat} = 0.23282\, V_{out1} - 2.22485\, V_{out2}$

$V_{bat} = k_3 (V_{out1} - V_{out2})$ (4)

$V_{bat} = 2.105\, (V_{out1} - V_{out2})$



Here is the measured data and the calculated results.

Vhigh	Vlow	Vbat	Ibat	Vout2	Vout1	calc Vbat	error, %	calc Ibat	error,%		
10.06	-0.010	10.07	20	0.550	5.34	10.08	0	20	-2	R1	0.502
10.05	-0.023	10.07	46	0.537	5.32	10.07	0	44	-4	R2	3900
10.02	-0.050	10.07	100	0.510	5.29	10.06	0	97	-3	R3	3970
9.96	-0.106	10.07	211	0.453	5.24	10.07	0	212	0	R4	464
9.92	-0.138	10.06	275	0.421	5.20	10.06	0	274	0	k1	0.23282
										k2	2.22485
										k3	2.10462

All voltages are with respect to ground except V_{bat}. V_{high} is the voltage at the positive terminal of the battery and V_{low} is the voltage at the negative terminal. V_{bat} is V_{high} - V_{low}. $I_{bat} = -\frac{V_{low}}{R_1}$.

I was careful to measure each resistor and record its actual value. With a large current flowing out of the battery, voltages were measured in a few places along each node. This identified voltage drops due to poor clip lead connections. At first my errors were greater than 20% but once all of these little errors were removed, my calculated V_{bat} error went to zero and my calculated I_{bat} was below 5%.

If you wish to be contacted each time I publish an article, email me with just "Article Alias" in the subject line.

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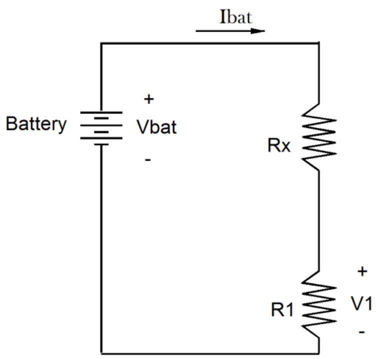
Appendix I: Measuring R_1



Measuring R_3 and R_4 is not difficult given a typical multimeter. But R_1 is too small for most meters. Instead, we can indirectly measure R_1 by passing a known large current through it and measuring the resulting voltage.

Say our multimeter has a 200 mV scale and we wanted to read 180 mV. Assume an R_1 of nominally 0.47 ohms. We would need to pass $\frac{180\ mV}{0.47\ ohms} = 0.38\ amps$ through R_1. Since 180 mV is much less than our battery voltage, we can estimate R_x as equal to $\frac{V_{bat}}{0.38\ amps}$.

For example, if $V_{bat} = 8.4V$, R_x would need to be around $\frac{8.4V}{0.38\ amps} = 22$ ohms. It would have to have a power rating of more than $\frac{(V_{bat})^2}{R_x} = \frac{(8.4)^2}{22\ ohms} = 3.2$ watts.

To measure R_1:

1. Calculate and select the needed Rx based on the battery voltage.
2. Temporarily connect R_x, R_1, and the battery in series.
3. Configure the multimeter to measure DC current[7] and put it in series with the battery but only connect one of the meter leads.
4. Momentarily connect the second meter lead and record I_{bat}.
5. Remove the meter and probes from the circuit.
6. Configure the multimeter to measure DC voltage and put across R_1.
7. Momentarily connect the battery and record the voltage V_1.
8. $R_1 = \frac{V_1}{I_{bat}}$

[7] This requires setting the dial to DC current and also moving one of the probe connects.

Appendix II: Alternate Method of Finding k_1 and k_2

This method derives the two constants from two measured data points so is exact at these points. When used on a third point, the error was 1.8%. This error is comparable to deriving the constants from measuring resistance values.

On page 3 I presented

$$I_{bat} = k_1 V_{out1} - k_2 V_{out2} \qquad (1)$$

Say I connect a resistor for the load and adjust the battery to give a current I_{bat1}. Then I measure V_{out11} and V_{out21}. Note that the first digit indicates which voltage and the second digit indicates which test phase.

I can then write for test phase 1

$$I_{bat1} = k_1 V_{out11} - k_2 V_{out21} \qquad (a1)$$

Next change the battery voltage by around 50%. Repeat the measurements to get

$$I_{bat2} = k_1 V_{out12} - k_2 V_{out22} \qquad (a2)$$

for test phase 2.

Using algebra, I can solve for k_1 and k_2:

$$k_4 = V_{out22}V_{out11} - V_{out21}V_{out12} \qquad (a3)$$

$$k_1 = \frac{V_{out22}I_{bat1} - V_{out21}I_{bat2}}{k_4} \qquad (a4)$$

$$k_2 = \frac{V_{out12}I_{bat1} - V_{out11}I_{bat2}}{k_4} \qquad (a5)$$

Where current is in amps. The constants are in $\frac{1}{ohms}$.

Let's test these equations with the measured data:

$$k_4 = 0.453\ X\ 5.29 - \ 0.510\ X\ 5.24 = \ -0.27603 \qquad \text{(a3)}$$

$$k_1 = \frac{0.453\ X\ 0.100 - \ 0.510\ X\ 0.210}{k_4} = \frac{-0.06180}{-0.27603} = 0.22389 \qquad \text{(a4)}$$

$$k_2 = \frac{5.24\ X\ 0.100 - \ 5.29\ X\ 0.210}{k_4} = \frac{-0.58690}{-0.27603} = \ 2.12622 \qquad \text{(a5)}$$

Going back to (1) and plugging in these constants yields

$$I_{bat} = \ 0.22389 V_{out1} - \ 2.12622 V_{out2} \qquad \text{(1)}$$

Using our first data point:

$I_{bat} = \ 0.22389\ X\ 5.29\ - \ 2.12622\ X\ 0.510$ = 0.10001 amps ≈ 0.100 amps.

Note that the constants were not rounded off but the final value was rounded to 3 places. This was done to avoid round off error. Since the constants were derived from these measured values, seeing 0.10001 rather than 0.10000 is due to round off error. Since the final answer was rounded to 3 places, this error does not matter.

Using our second data point:

$I_{bat} = \ 0.22389\ X\ 5.24\ - \ 2.12622\ X\ 0.453$ = 0.21001 amps ≈ 0.210 amps.

As an overall check, let's use these *k* values for a third data point:

$I_{bat} = \ 0.22389\ X\ 5.20\ - \ 2.12622\ X\ 0.421$ = 0.26909 amps ≈ 0.270 amps.

The measured current was 0.275 amps so the error in the equation is 1.8%.