

Elmer's Flywheel

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Background



My favorite steam engine book was written by the Elmer Verburg and is called *Elmer's Engines*². In Chapter 24 he presents a wonderful little beam engine with a graceful flywheel. The intent here, as with most of his designs, is to just enjoy yourself and not get all wrapped up in complex machining. It is possible to make this flywheel by eye and it comes out fine. I've done it as you can see here. But for me the greater challenge is to understand how to calculate the location of each cut of the spokes as a means of mastering the Rotary Table (RT). I do hope that Elmer's spirit will be understanding.

When done, I tested out the math on a piece of scrap wood.



Conclusion

With the Rotary Table's center of rotation aligned with the mill table's (0,0) and the angle set to 0 along the positive X axis, the spokes of the flywheel can be cut by using the following values:

RT angle of +/- 3.7 degrees centered at 0 degrees and incremented 60 degrees at a time

Y value of +/- 0.1805" (assuming an 1/8" diameter end mill)

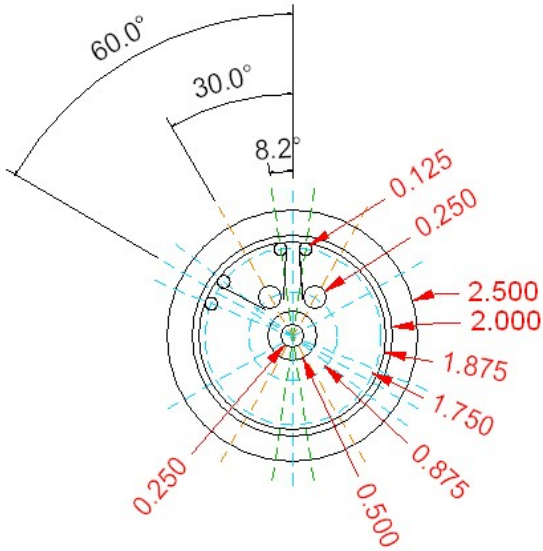
X value starting at 0.364" and going to 0.856"

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² ISBN 0-9621671-0-X

Elmer's Design

The flywheel is cut from 2.5" diameter stock. I'm using 12L14 leaded steel but aluminum or brass would work too. A recess is cut in both faces that starts at a diameter of 2" and goes in to a diameter of 0.5". At 0.5" we have the hub with a 0.25" hole in it.



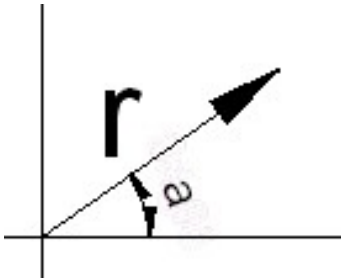
Offset 30 degrees and spaced every 60 degrees on a 0.875" circle we have 0.25" holes. Out at a diameter of 1.750" we have a series of 0.125" holes. These holes are offset +/- 8.2 degrees from a center-line between the 0.25" holes. Two 0.125" holes and a 0.25" hole define the corners of a triangle shaped area that is milled out. The result is a series of tapered spokes that look great. The challenge is figuring out the correct RT angle plus XY location to cut the sides of these spokes.

The simple way to do this task is to put a square on the RT referencing its base. Then turn the RT until the square lines up on the tangent of a 0.25" and 0.125" hole. This works fine if the goal is to just cut the spokes. But it doesn't teach me how to use the RT.

Geometry!

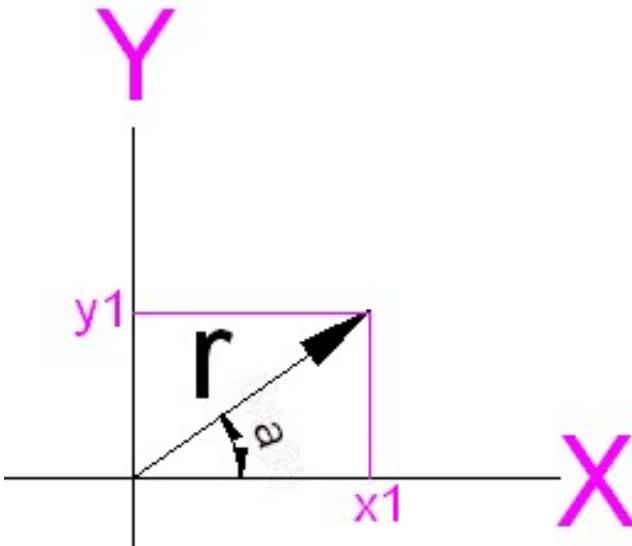
I'm sure there are many ways to solve this problem. I've chosen to use geometry. No higher math here but there may be things that you have forgotten over the years. Lets review a few key concepts.

The first is **polar notation**.



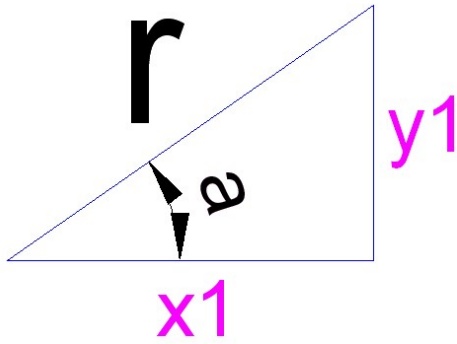
I can locate a point as being at some radius from my origin and at some angle from a defined 0 degrees. This is standard practice on an RT.

Now lets look at **rectangular notation**.



Rather than move my RT, I can move my mill table to a point at x_1, y_1 and get to the same place as turning my RT “a” degrees and then moving out a distance r .

We get to the same point with both methods. This means that given that I know the location of a point using one coordinate system, I can calculate that position with the other coordinate system. Time to drag in a bit of trigonometry. Please try to resist the urge to run from the room.



I have drawn a triangle with both polar and rectangular notation on it. The hypotenuse has a length “r”. The base is “x1” long and the height is “y1” tall. I also have an angle “a”. All of these parameters are tightly related.

(Equation 1) $r^2 = (x1)^2 + (y1)^2$ which says that the square of the hypotenuse equals the sum of the the square of the base and the square of the height.

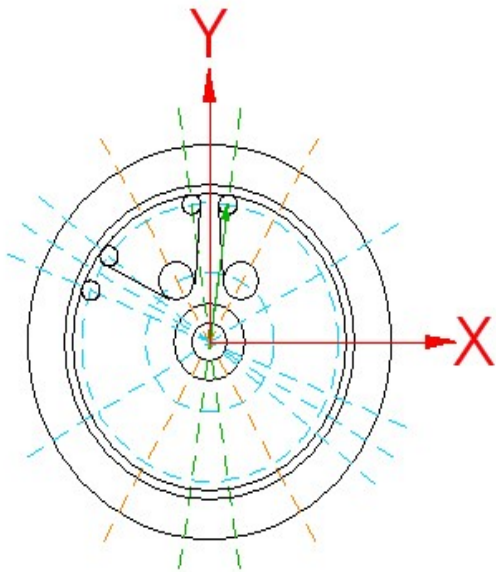
(Equation 2) $y1 = r * \sin(a)$ which says that my height, y1, equals the hypotenuse times the sine of the angle “a”.

(Equation 3) $x1 = r * \cos(a)$ which says that my base, x1, equals the hypotenuse times the cosine of the angle “a”.

(Equation 4) $a = \sin^{-1} (y1/r)$ which says that my angle “a” equals the arc sine of the height divided by the hypotenuse and is a rearrangement of Equation 1.

(Equation 5) $a = \cos^{-1} (x1/r)$ which says that my angle “a” equals the arc cosine of the base divided by the hypotenuse and is a rearrangement of Equation 2.

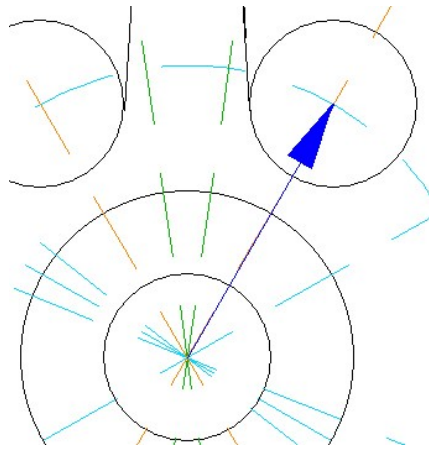
If these equations are not familiar, not to worry. As we use them they will make more sense.



Using the New Math Tools

The first thing I must do is define my two coordinate systems such that they share the same origin. As part of my RT set up on my mill, I have set (0,0) at the center of the RT. My flywheel blank is also centered at this point.

Looking down from the top, I have my RT's angle set to 0 aligned with my positive X axis. Clockwise rotation causes the angle to increase.



Here is a close up of the flywheel. The distance from the center to the 0.25" hole is $0.875''/2 = 0.4375''$. The angle with respect to the vertical is 30 degrees. From the horizontal, our 0 degree reference, the angle would be $90 - 30 = 60$ degrees.

I have drawn the radius and angle using rectangular notation.

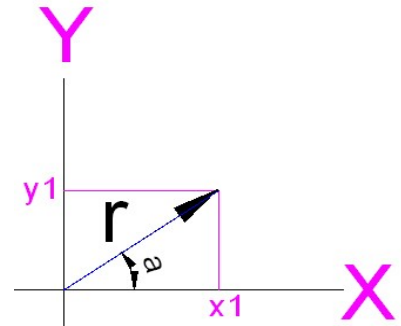
Using Equation 2 I can say

$$y1 = r * \sin(a)$$

$$y1 = (0.4375'') * \sin(60 \text{ degrees})$$

$$y1 = 0.379''$$

Similarly, with Equation 3 I can calculate that $x1 = 0.219''$



As a check of my math, I can use Equation 1 and verify that $r^2 = (x1)^2 + (y1)^2$

$$r^2 = (x1)^2 + (y1)^2$$

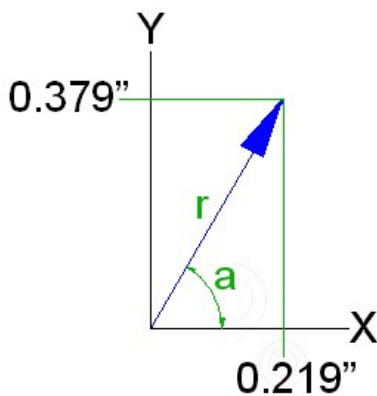
$$r^2 = (0.4375'')^2$$

$$r^2 = 0.191 \text{ square inches}$$

$$(x1)^2 + (y1)^2 = (0.219'')^2 + (0.379'')^2$$

$$= 0.192 \text{ square inches}$$

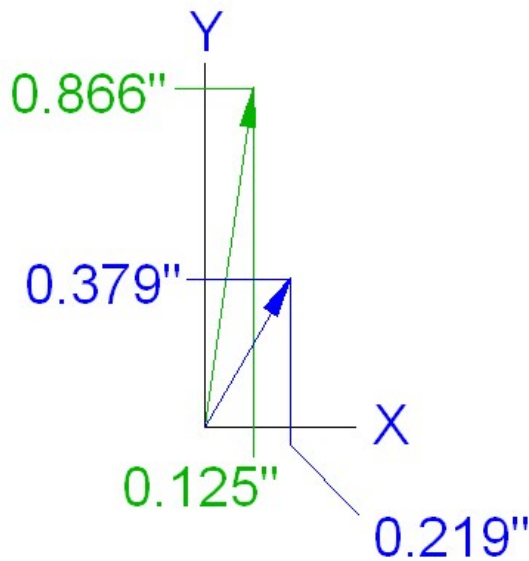
The difference in answers comes from round off error because I limited all numbers to 3 places except for x1.



Here we have the location of that 0.25" hole with respect to the center of the RT using rectangular notation.



I can do the same thing for a nearby 0.125" hole. The tangents between these two holes will be the line I wish to mill. The green arrow locates my 0.125" hole. It is at a radius of 0.875" from the center of the RT and at an angle of 81.8 degrees from horizontal. Converting to rectangular notation, this means I am at X = 0.125" and Y = 0.866".



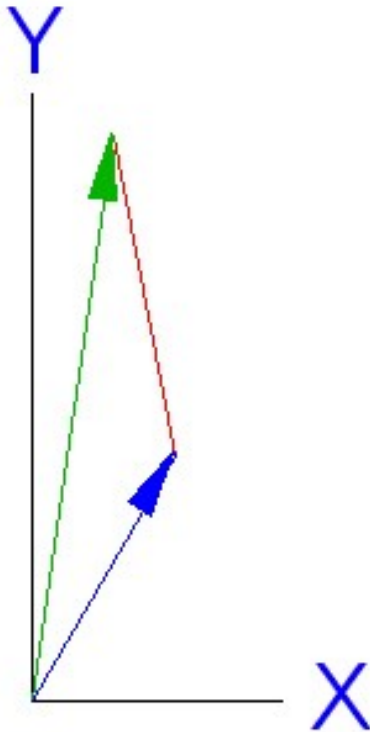
I have put the location of the 0.25" hole and of the 0.125" hole on the same graph.

Consider just the X axis. The X axis distance between these two holes is $0.219'' - 0.125'' = 0.094''$. Next, just look at the Y axis. The Y axis distance between these two holes is $0.866'' - 0.379'' = 0.487''$. What we have just done is called Vector Arithmetic. We subtracted the distance between two points in rectangular notation. It is also called the XY plane.

I will now use these X, Y values.

This red line is the hypotenuse of a triangle with a base equal to 0.094" and a height of 0.487". Equation 1 tells me that the hypotenuse is 0.496" long. Equation 4 tells me that the angle at the bottom right is 79.1 degrees.

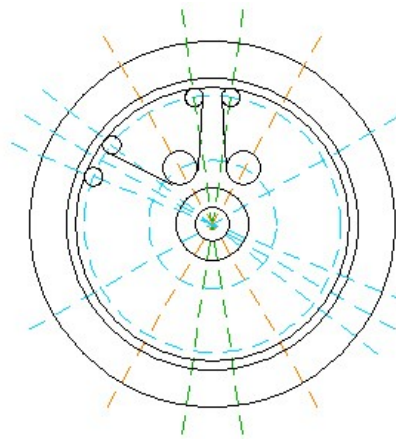
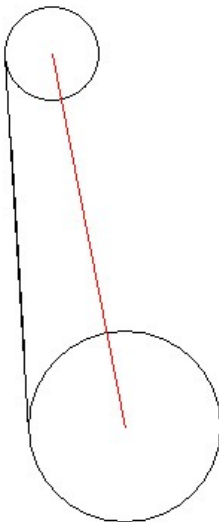




This red line is at the correct angle and length for the base and height I calculated from my vector subtraction. If I don't change its angle or length, I can move this line anywhere and not change its value. OK, I choose to move it to the tips of the two arrows. It will fit perfectly because it is, in fact, the difference between the green vector and the blue vector. Vector is just another name for my radius and angle.

I know a lot about this red line. For example, since I know the location of the tip of the blue vector, I know where the red line starts. I know the length of the red line and I even know that the angle of the line is 79.1 degrees from the horizontal as shown on the last page. The other vectors measure their angle from the X axis so my red line is at an angle of $180 - 79.1 = 100.9$ degrees from that axis.

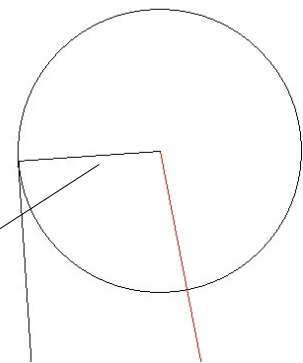
In other words, I know exactly where that red line is located. Next, let's draw back in the two holes.

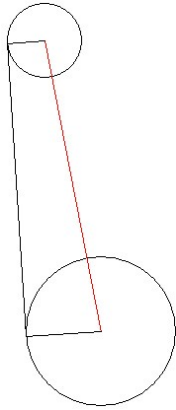


You can now see how the red line connects the centers of the two circles. That black line that goes between the holes is the path we want to follow with our cutter. The goal is to define this line so we can correctly set up the RT angle and XY of the mill table.

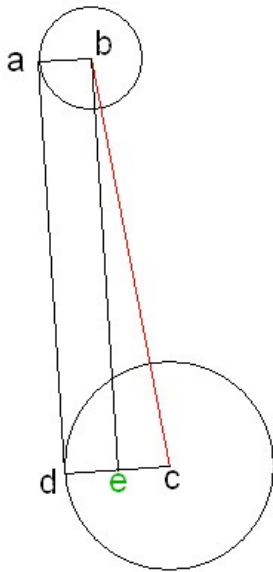
Our next challenge is to relate the black line's location which we don't know to the red line which we do know.

We do know a few more bits of information here. The small circle has a diameter of 0.125" and the large circle has a diameter of 0.25". We also know that the black line is tangent to both circles. Geometry tells us that a line from the center of a circle to a point on the circle will be perpendicular to any tangent from that point.





We now know all about the red line plus the length of the two radii that are perpendicular to the black tangent line.



Let me label the various points to keep them straight. Because both lines a-b and c-d are perpendicular to a-d, I know that a-b is parallel to c-d.

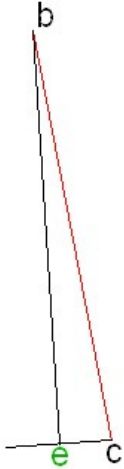
I can draw a line from point b down to line c-d such that b-e is perpendicular to c-d. Picture sliding line a-b down line a-d until it flanks line c-d. Line a-b will be the same length as line e-d. Line b-e must be the same length as a-d because I have created a rectangle a-b-e-d and b-e is opposite a-d.

Since we know the length of line a-b and line c-d, we know the length of line c-e is just the length of c-d minus the length of a-b.

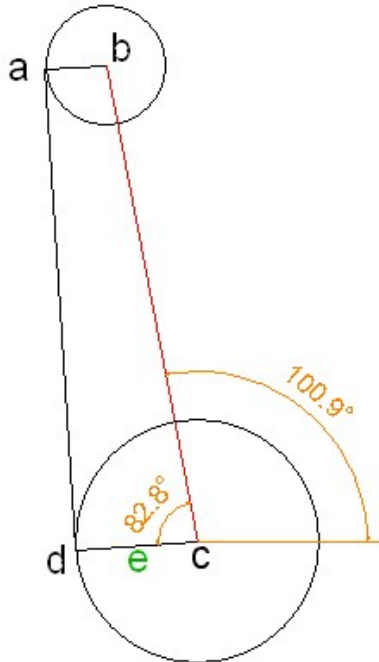
We know the length of c-b, and c-e. Using Equation 1 we can find the length of b-e which is also the length of a-d which is the length of the cut to be made by my mill.

$r^2 = (x1)^2 + (y1)^2$ and we know $(\text{line } b-c)^2 = (\text{line } c-e)^2 + (\text{line } b-e)^2$. Plug in our numbers and we have $(0.496'')^2 = (0.0625'')^2 + (\text{line } b-e)^2$. Solving for line b-e we square 0.496 and subtract from it the square of 0.0625. Then we take the square root. This gives us a length of 0.492'' for line b-e which is also the length of line a-d.

We need one more piece of information: the angle of line a-d.

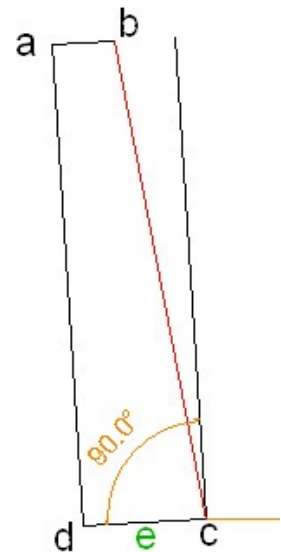


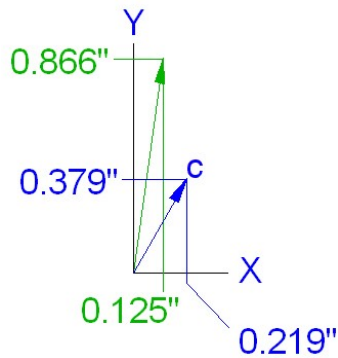
Look at the triangle formed by points b-c-e. We know the lengths of sides b-c and c-e so can use Equation 3 to find the angle at point c: $\arccos(\text{line c-e}/\text{line b-c}) = \arccos(0.0625/0.496) = 82.8$ degrees.



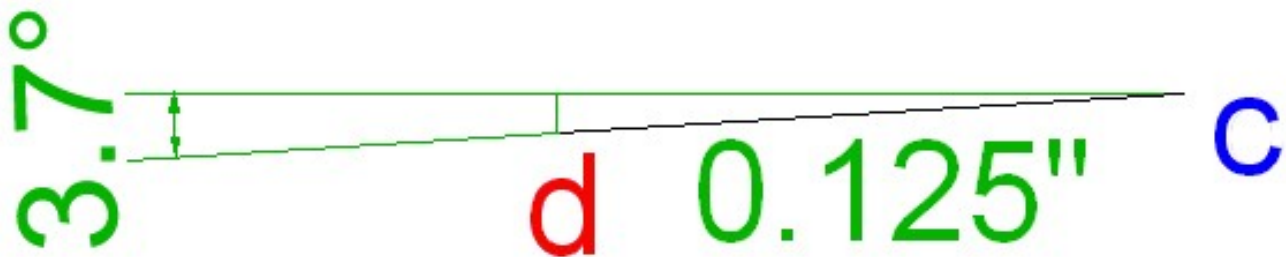
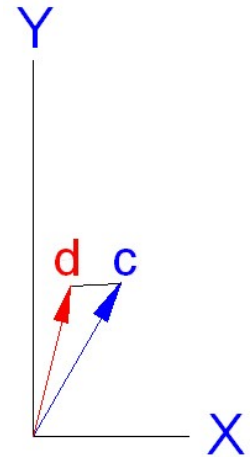
We are now ready to put this all back together. The red line b-c is at an angle of 100.9 degrees from or 0 degree reference. We found this from subtracting the vector to point b from the vector to point c. The angle inside the triangle at point c is 82.8 degrees. We found this from understanding triangle b-c-e. Adding these two angles together gives us the angle of line c-d with respect to the 0 degree reference. It is $100.9 + 82.8 = 183.7$ degrees.

I know that line a-d is perpendicular to line c-d but it isn't that clear what to do about the angle. The answer is to draw a line parallel to a-d that passes through point c so all angles are centered on the same point. I can now calculate the angle of line a-d with respect to my 0 degree reference: $100.9 + 82.8 - 90 = 93.7$ degrees. If I want the angle of line a-d with respect to the vertical, I subtract another 90 degrees and get 3.7 degrees.

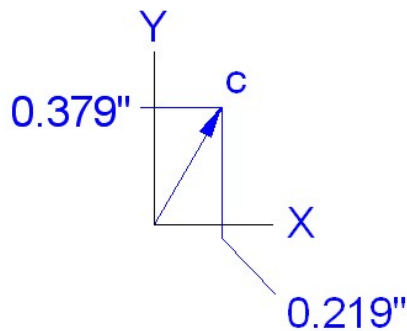




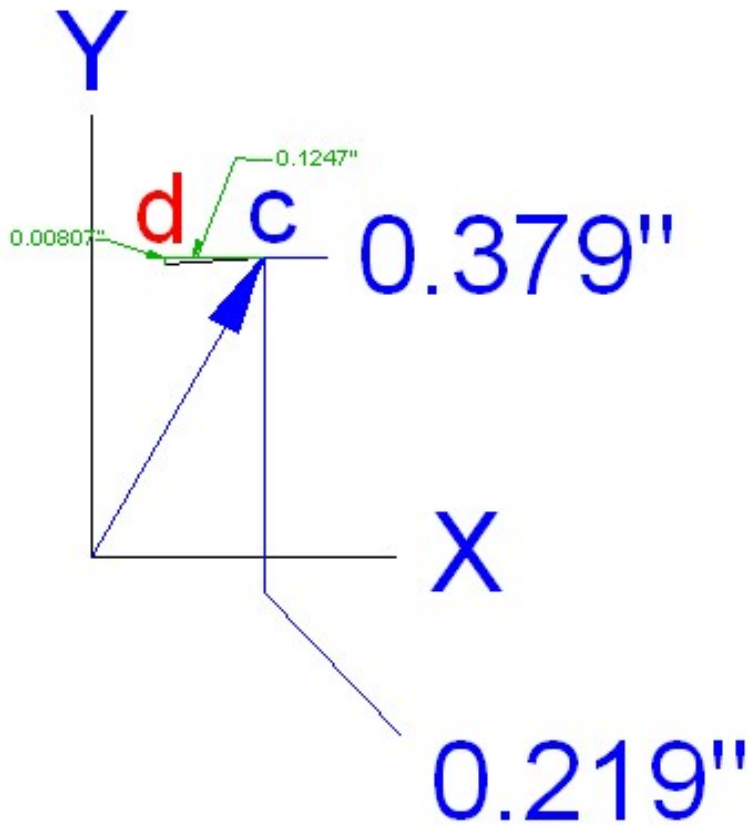
Back on page 6 you saw this blue vector that locates what we now call point "c", the center of the 0.25" circle. We want to find the location of point d. To do this, we need to add the point c vector to the vector from points c and d. Time to take a close look at vector d-c.



Back on page 9 we found that the line c-d was at an angle of 183.7 degrees with respect to the 0 degree reference. We want to know this angle with respect to the negative X axis so subtract 180 degrees. That leaves us an angle of 3.7 degrees. Line c-d is 3.7 degrees below the negative X axis. Using Equation 2 we find that the height of this triangle is $(0.125") * \sin(3.7 \text{ degrees}) = 0.00807"$. Equation 3 gives us the base: $(0.125) * \cos(3.7 \text{ degrees}) = 0.1247"$.



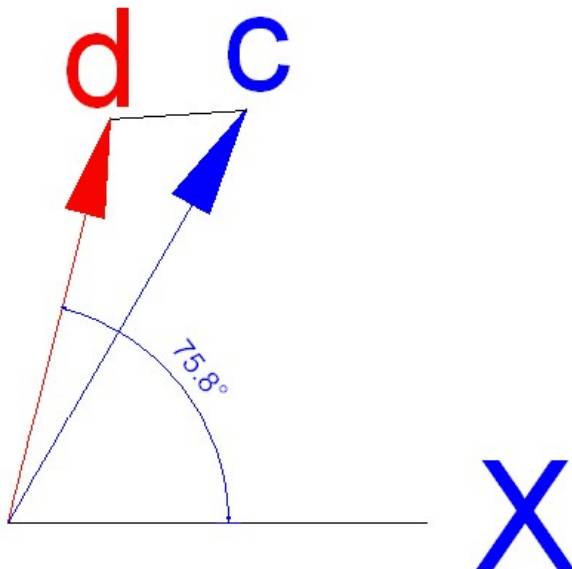
On page 6 we subtracted two vectors. This time we will add two vectors. Vector c was found to point to the location $X = 0.219"$ and $Y = 0.379"$.



Now I have added vector d-c. Point d is at an X value of $0.219'' - 0.1247'' = 0.0943''$ and a Y value of $0.379'' - 0.00807'' = 0.3709''$.

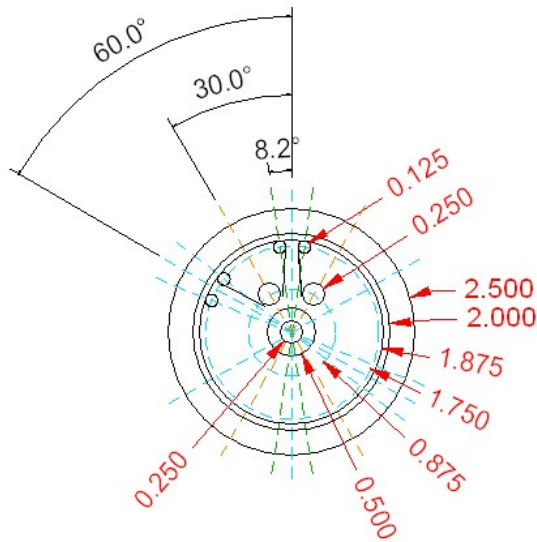
A minute ago I said I was adding these vectors but I just subtracted them. The confusion comes from the fact that vector d-c points in a negative direction, down and to the left. So I really have a change in X of $-0.1247''$ and in Y of $-0.00807''$. If the vector c-d points up and to the right, both numbers would be positive.

So now I know that the start of my cut, point d is at $X = .0943''$ and $Y = 0.3709''$. Converting back to polar notation we get a radius equal to the hypotenuse (via Equation 1) of $0.383''$. We can use Equation 2 to find that the angle is 75.7 degrees.

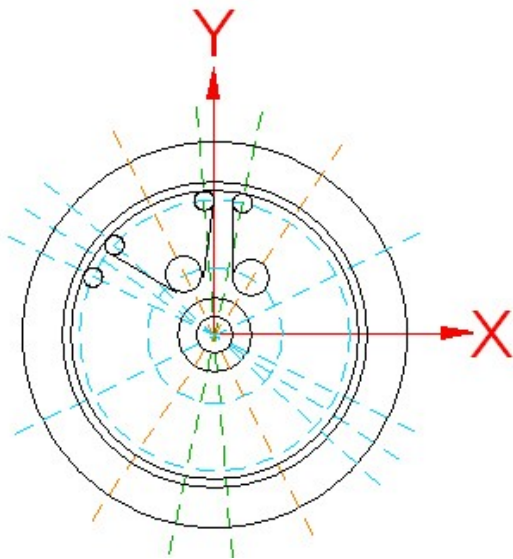


I converted back to polar because I need to locate point d after I rotate my RT by 3.7 degrees. With polar notation, all I need to do is change the angle of the d vector and then convert back to rectangular notation.

Note that my calculations gave me 75.7 degrees but my CAD program gave me 75.8 degrees. I suspect that all of that rounding of values in my calculations caused this difference. The CAD answer is probably closer to the truth.



Returning to the original drawing, we can now apply the numbers we worked so hard to find.



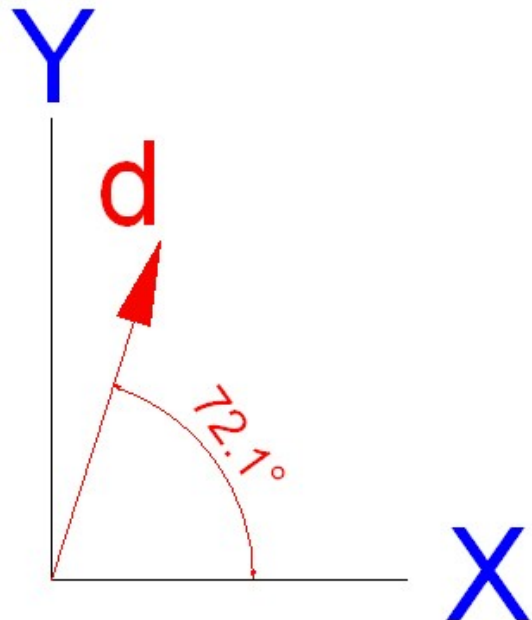
First of all, the RT must be turned 3.7 degrees clockwise in order to make a cut that defines the right side of a spoke. This rotation puts the right side of the spoke parallel with the Y axis of my mill.

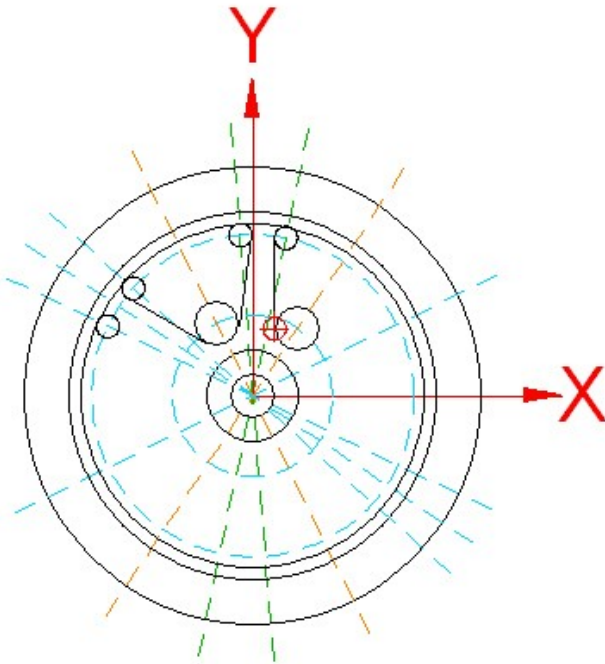
From page 11 we learned that the end of the cut nearest the center of the RT is at a radius of 0.383" and an angle of 75.7 degrees. When we rotated the RT counterclockwise by 3.7 degrees, our angle must be decreased by 3.7 degrees. So after rotation, this point is at an angle of $75.7 - 3.7 = 72.0$ degrees. That 0.1 degree discrepancy between my calculations and my CAD program shows up again as the angle is 72.1 degrees in the figure.

Converting back to rectangular notation, we have

$$X = (0.383'') \times \cos(72.0 \text{ degrees}) = 0.118''$$

$$Y = (0.383'') \times \sin(72.0 \text{ degrees}) = 0.364''$$

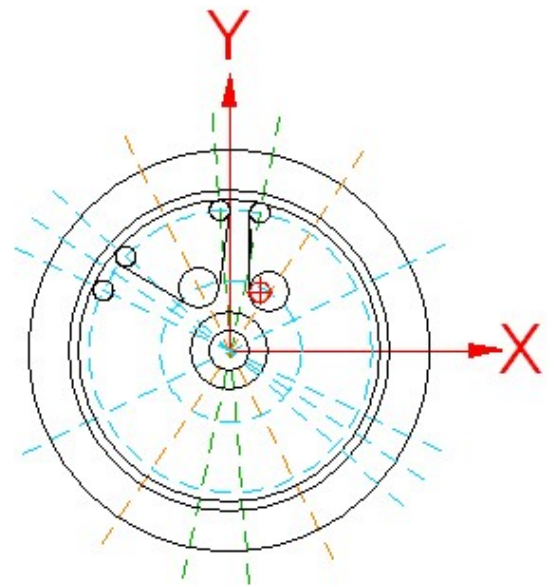




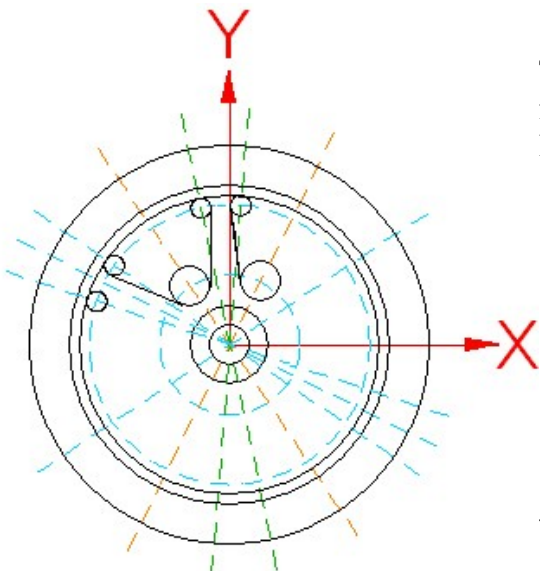
With the RT rotated 3.7 degrees clockwise, I must now move the mill table from (0,0) at the center of the flywheel to my point "d" which is at $X = 0.118''$ and $Y = 0.364''$. The red circle with the cross in it is this location.

Before we can power up the mill, one more calculation must be made – tool offset. Point "d" is the start of the cut but I must shift over by the radius of the end mill. My end mill is 0.125" in diameter so I want to shift the cutter along the X axis by half of this value.

This puts my cutter at $X = 0.118'' + 0.0625'' = 0.1805''$.
Y remains at 0.364".

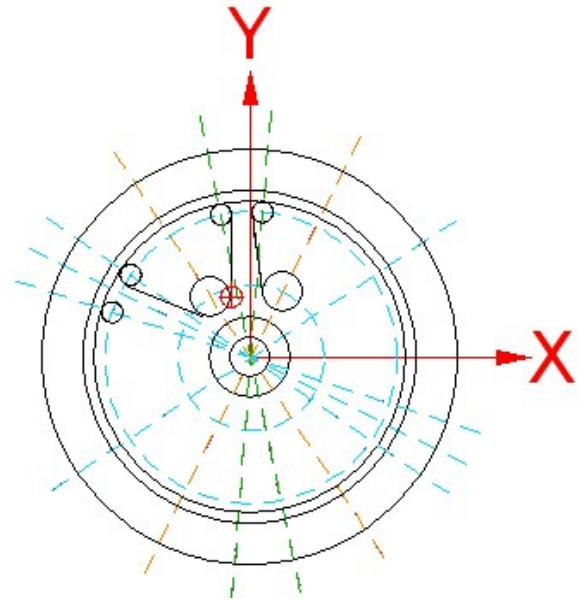
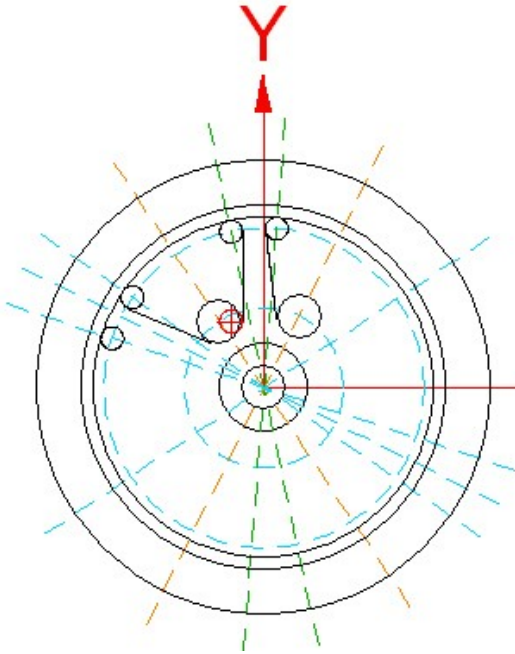


From page 8 we learned that the line is 0.492" long . We therefore must start our cut at $X = 0.1805''$, $Y = 0.364''$ and end at $X = 0.1805''$, $Y = 0.364'' + 0.492'' = 0.856''$.



The left side of the spoke is the reverse of the cut we just finished. We first move the RT to -3.7 degrees. Now we have the left side of the spoke parallel with the Y axis.

We move the cutter to $X = -0.118''$ and $Y = 0.364''$.

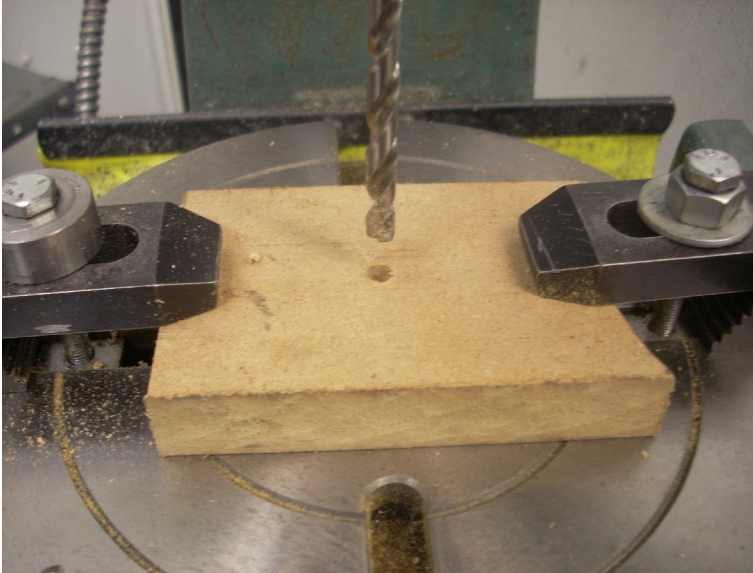


Don't forget the tool offset. X changes to $-0.1805''$. We then start the cut at $Y = 0.364''$ and stop at $0.856''$.

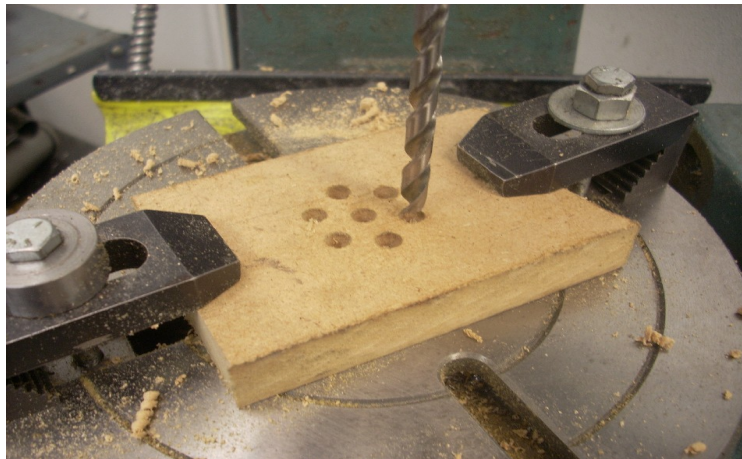
We have now cut one complete spoke. It would probably be faster to cut all of the right sides as a set. Then you are only moving the RT and the table along the Y axis. Then reposition X and cut all of the left sides in the same way.

Shop Work

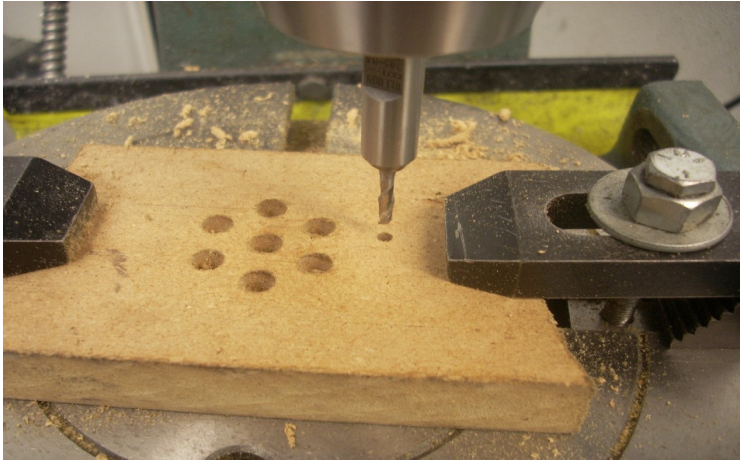
The math is nice but without proof that it works, I have wasted your time.



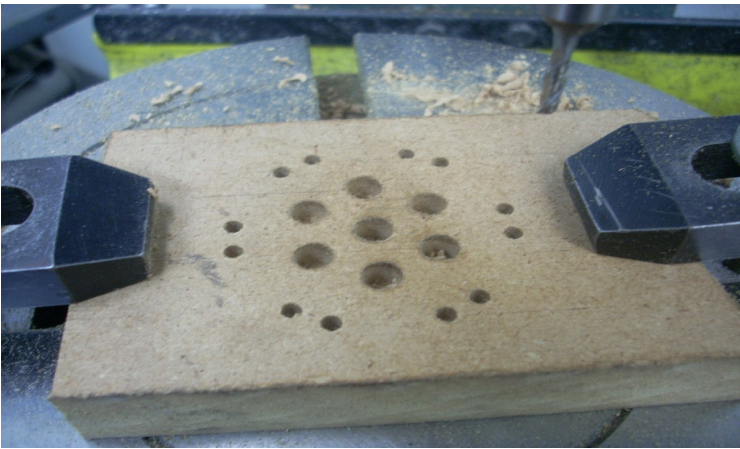
At this point, I have about as much faith in my numbers as you have so will test it out using a piece of scrap MDF. I have just drilled the 0.25" center hole.



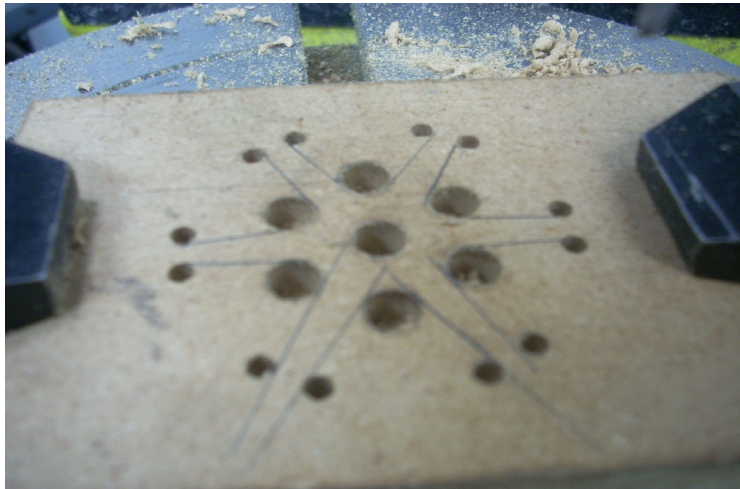
All six 0.25" holes were drilled next. $X = 0.4375''$ and $Y = 0$. The RT starts at 30 degrees and increments 60 degrees each time. The work goes fastest by leaving the mill running.



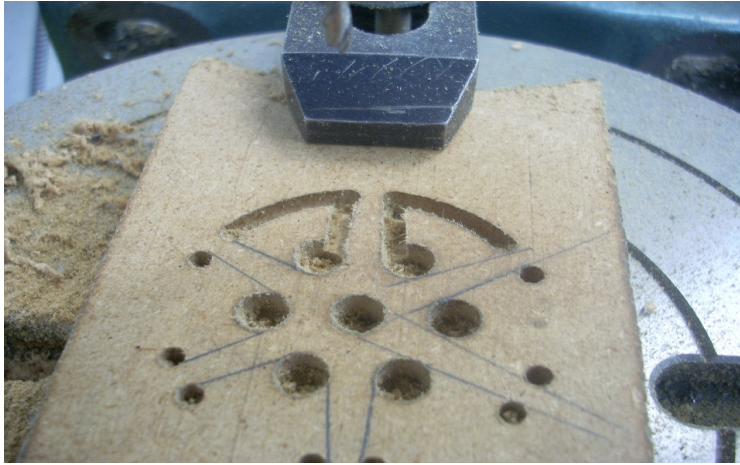
I have now moved to $X = 0.875''$ and $Y = 0$. I'm using a 2 flute $1/8''$ end mill to drill these holes. If I was cutting 12L14, I would use a regular drill to spare wear and tear on this end mill.



The RT will be moved to 0 first. I then move ± 8.2 degrees from this point. Advance to 60 degrees and again drill ± 8.2 degrees from this point. In no time I have drilled the 6 sets of holes.



I drew in rough layout lines to prevent me from making really big mistakes.



I cut my first spoke plus cut part of the perimeter. Notice the overshoot on the far end of the left flank of the spoke. I got one of the angles wrong. As with any project with so many steps, it only takes on mistake to screw up the part. I'll try another spoke.



It is hard to see, but I did get this spoke right. The work does go very fast.

I have one major point of confusion. In order to get my numbers to work right, I had to swap my X and Y values. I used $Y = \pm 0.1805''$ and X between $0.364''$ and $0.856''$. The layout lines made this obvious. At the moment I don't see what I did wrong. Hopefully one of my readers will see it. Then Version 2 will have it all right.

Acknowledgments

I would like to thank John Walker for his insights and encouragement. John also developed a spreadsheet that can give you the key data for any size flywheel. Thanks to Larry Gill for finding all of my typos and lapses in clarity.

I welcome your questions and comments. They will be used to improve this document. All of us are smarter than any one of us.

Rick Sparber
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