

Designing a Project Box, Version 2.5

By **R. G. Sparber**

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Scope



If you just want to bend up boxes, download the Excel spreadsheet² at

<https://rick.sparber.org/BoxBending.xls>

and follow the directions that start on page 3.

If you want to understand how I came up with these equations, you can read as deep into the article as they wish. It does get a bit hairy in the variational analysis section. You may need to read

<https://rick.sparber.org/SheetMetalBending.pdf>.

first, to understand the terminology and basic concepts.

The article is as much about my journey as it is about the destination. I will show you how I approached the problem of creating the equations plus laying out the sheet metal that will fold to become a box.

Dedication

I never met Wayne Smith's father. He had a successful career bending sheet metal with high precision. Knowing this was enough to inspire me to learn about the subject, derive a few equations, and write this article.

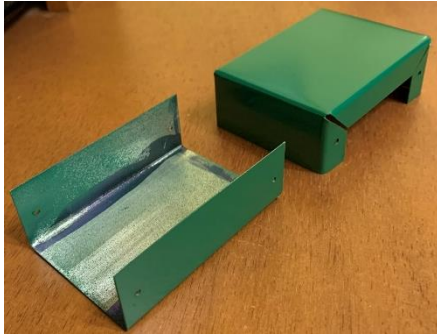
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² Depending on your security settings, you may need to right click on the file name and select Properties, At the bottom of the window is an Unblock check box. Click it, close the window, and you should be able to open the spreadsheet.

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A Physical Model



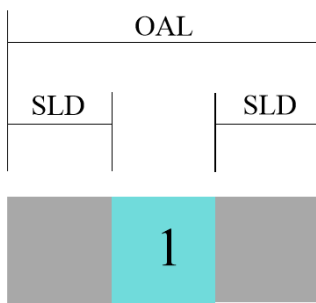
A physical model is a great way to help me not get lost. Even if my model was made from paper, it would still be invaluable. This is the finished box, which is a lot easier on the eyes than the one I used as my model.

I will check back with it as I draw models of ever-increasing complexity.

I Just Want to Bend Boxes

You may want to jump directly to the spreadsheet, which will translate your requirements into the layout on sheet metal. But first, the spreadsheet needs to know a bit about your brake, the sheet metal you plan to use, and your skill.

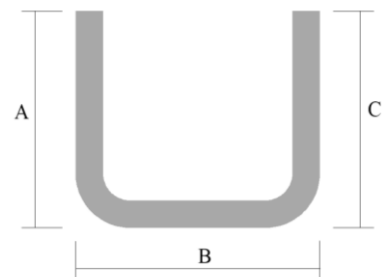
Characterizing Your Brake and Your Skill



Before you can accurately bend sheet metal, you must characterize your brake with the material to be bent. You must also characterize your marking out skill and bending skill. You do this by making a series of test bends from the selected material and recording the resulting measured data in the spreadsheet.

1. Cut five test strip 3.000-inches long with straight sides and square corners.
Do not cut one rectangle 3 inches wide and then slice into strips.
2. Label the strips 1 through 5.
3. To the nearest 0.001-inches, measure the OAL of each strip and record it.
4. Scribe a sight line 1.000-inches from each end of each test strip (SLD).
5. With the short end under the fingers of the brake, bend a 90° at each end.
6. Measure and record A, B, and C for each strip.

This data must be put into the spreadsheet, which is described next.



Using the Spreadsheet

Each block of cells is marked with [n] where *n* goes from 1 to 6.

[1] Characterize your brake with the selected sheetmetal

| sample | measured OAL | measured A | measured B | measured C |
|--------|--------------|------------|------------|------------|
| 1 | 3.005 | 1.018 | 1.086 | 1.035 |
| 2 | 2.992 | 1.052 | 1.035 | 1.033 |
| 3 | 2.980 | 1.037 | 1.024 | 1.041 |
| 4 | 3.003 | 1.026 | 1.046 | 1.028 |
| 5 | 2.986 | 1.032 | 1.021 | 1.030 |

[1] This is where you input measurement from the five test bends. All measurements must be within ± 0.001 -inches to get the best fit.

[2] Characterize your selected material

| | |
|----------------------|-------|
| material thickness = | 0.020 |
|----------------------|-------|

[2] Input the thickness of the material used for the test bends, which must also be the material to be used to make the box.

[3] Define needed internal dimensions of the box

| | |
|----------|-------|
| height = | 1.000 |
| width = | 1.000 |
| length = | 1.000 |

[3] Input the inside dimensions of the box you wish to make. The resulting dimensions will not be less than these numbers.

[4] inside part:

| | |
|----------|-------|
| SLD = | 1.003 |
| width = | 3.057 |
| length = | 1.005 |

[4] The width and length of the sheet metal used to make the bottom part of the box are listed along with the sight line distance. The diagram to the right of these cells shows the layout.

[5]

| | |
|-------------------|-------|
| chosen C = | 0.500 |
| measured A = | 1.035 |
| measured B = | 2.085 |
| measured length = | 3.006 |

[5] You choose the width of the tab that flanks the sides of the box. This is "C." Then you measure "A" and "B" of the part you just bent up along with its length.

[6] outside part:

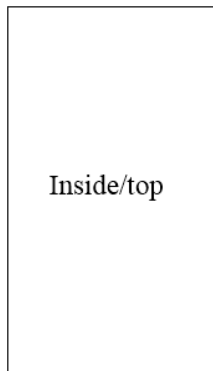
| | |
|----------|-------|
| length = | 5.132 |
| SLD L = | 1.064 |
| width = | 3.172 |
| SLD w = | 0.492 |

[6] The dimensions of the outside part are given. See the diagram to the right of these cells for the layout. See page 39 for the notches.

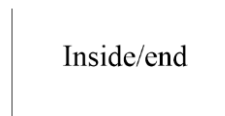
The Thought Behind the Spreadsheet

You do not need to know how and why the spreadsheet works if your focus is on just making boxes. But if you want to understand why and how it works, read on.

Starting Simple

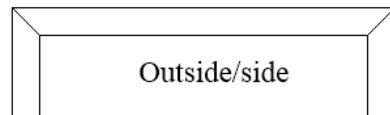
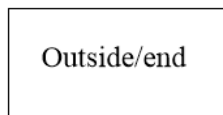


The simplest part of the box is the inside, so I started there. As I looked down on the inside part of the box, I saw this rectangle.

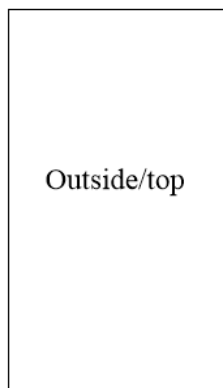


When I looked at the end, I saw this U shape.

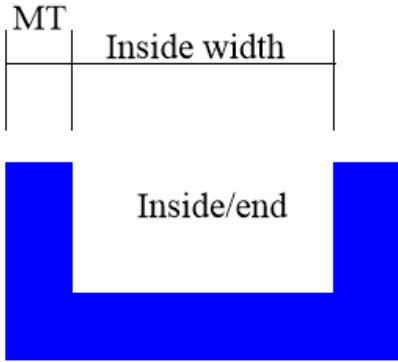
Ready for a bit more complexity?



The outside part wraps around the inside part. By looking at my physical model, I drew these figures.



Add Thickness and Define Specifications

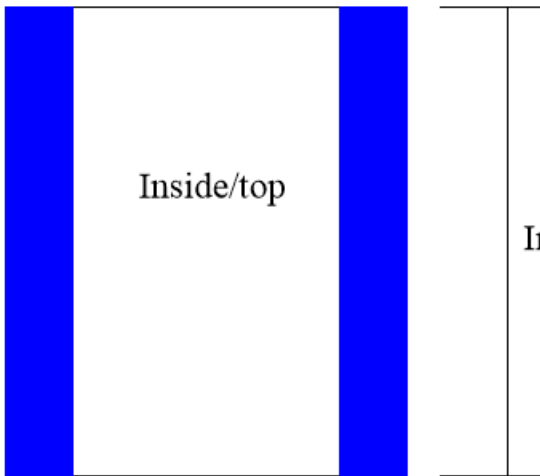


My drawings helped me see relationships. I have drawn the material thickness (MT) way out of scale so I can see how it impacts the design. I also added color to help me not get lost.

As I was drawing this figure, it reminded me of what is important: the inside dimensions.

I'm building the box to enclose something. So, here is my first design specification:

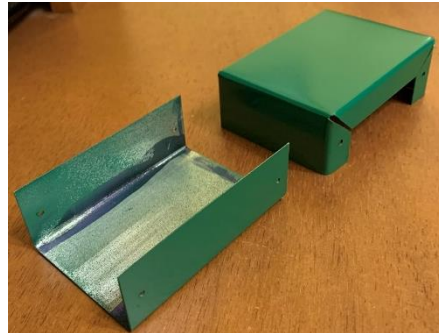
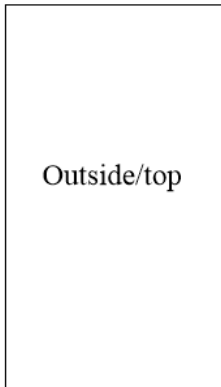
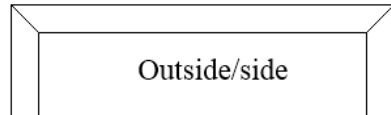
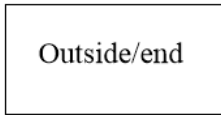
inside width.



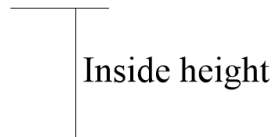
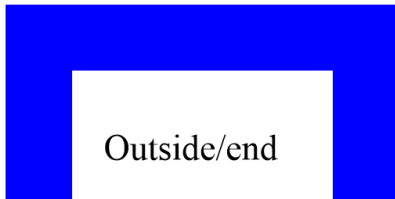
It wasn't much of a jump to realize I want to define

Inside length

as my second design specification.



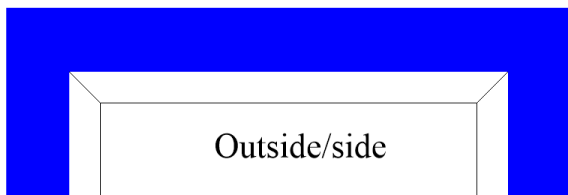
It helps me to review my physical model and the simplest drawing of the outside part before moving on.



The Outside/end gets larger as I add material thickness (blue). This is an X-ray view.

My third design specification is the **inside height.**

Adding thickness has made the drawing a little bit more confusing. When I start to get lost, I pick up my physical model, and my head clears.



The complexity of my outside/side drawing increases as I add material thickness. Remember, this is an X-ray view and is not in proportion.

All three of these specified dimensions are to ensure that what I put inside the enclosure will fit. There may be some extra room, but I should never be less than the specified height, width, or length.



I had to return to my physical model to confirm that the material thickness went all the way around.

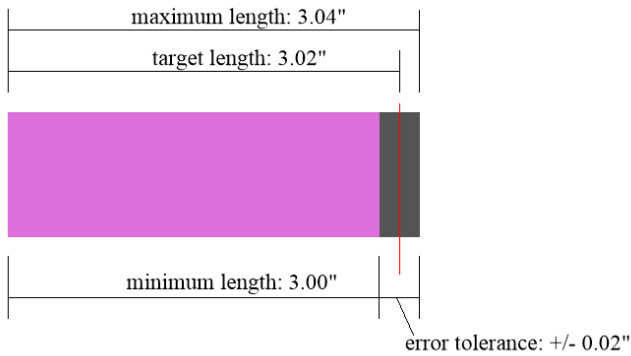
Outside/end

Outside/top



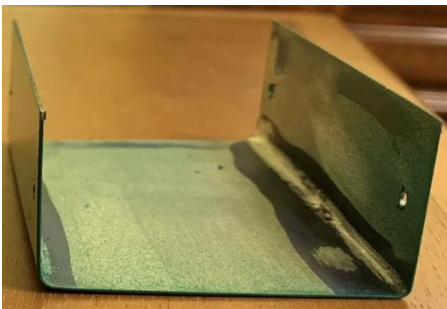
So far, we have only talked about perfect dimensions. Each cut and bend has a unique error tolerance associated with it. I must look at each dimension and allocate space for its error tolerance.

Error

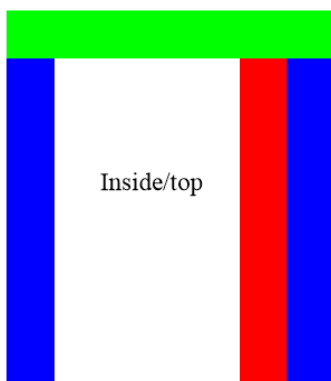
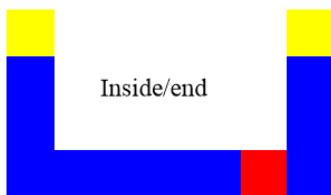


For example, say I want to cut a length to 3.000-inches but not less.

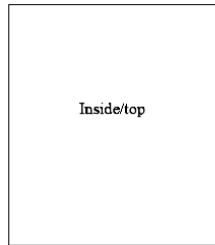
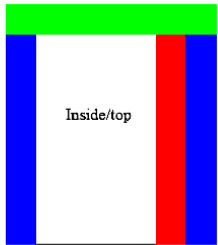
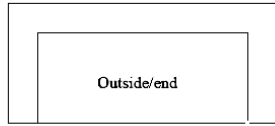
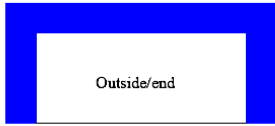
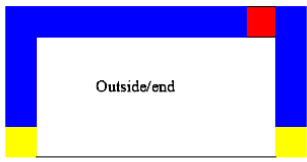
From past experience, I know that my cuts will be ± 0.02 -inches. To ensure that the length is never less than 3.000-inches, I will shoot for 3.020-inches. Then, in the worst case, if I'm under, I'll be at 3.000-inches. It might be as long as 3.040-inches.



Returning to the inside part, end view, we have one error tolerance associated with the sides (yellow) and a different error tolerance for the bottom (red). The yellow error is due to bending the metal while the red error is due to cutting and bending error.



A third error tolerance is associated with the length (green). This is due to only cutting error.



The outside part can get taller (yellow squares) and wider (red square) due to error. It will be at its smallest when these errors are at their minimum. In this state, it must still fit over the inside part.

All of these errors sure are confusing! By finding what is important to know before we move on, we get back to simple outlines.

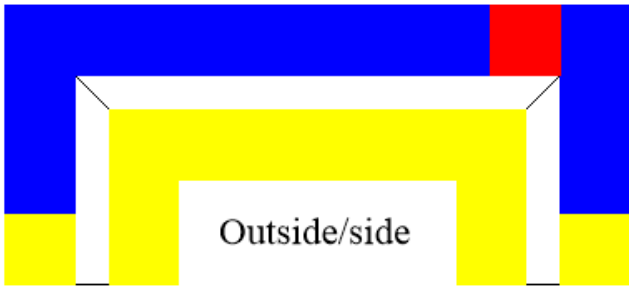
I'll bring back in the tolerances as needed.



Recall that I started with specified dimensions as I bent up the inside part. When finished, I measured the part and generated a new set of dimensions, which were then used to design the outside part. This strategy minimized the build up of error for the outside part.

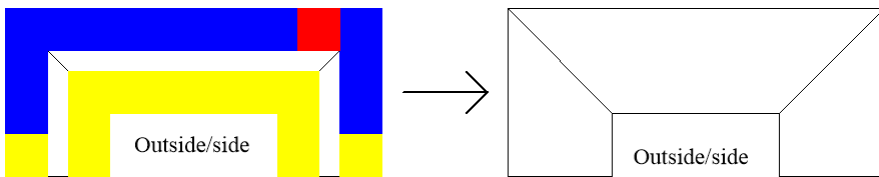


The outside part has the same errors associated with the top face (red) and sides (yellow). Remember, this is an X-ray view of the material thickness.



Because there are two more bends visible on the side view, we pick up another red error.

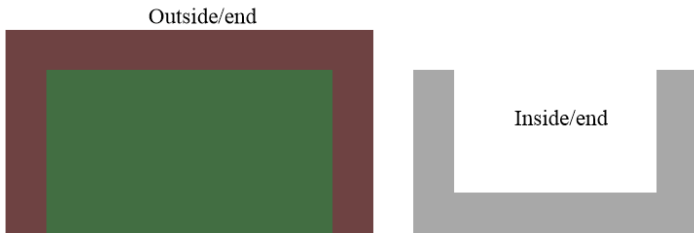
Using the same reasoning as was used for the top view, I know that when the inside part's measured length becomes a specification for the outside part. Knowing this, I can return to just a silhouette of the outside part.



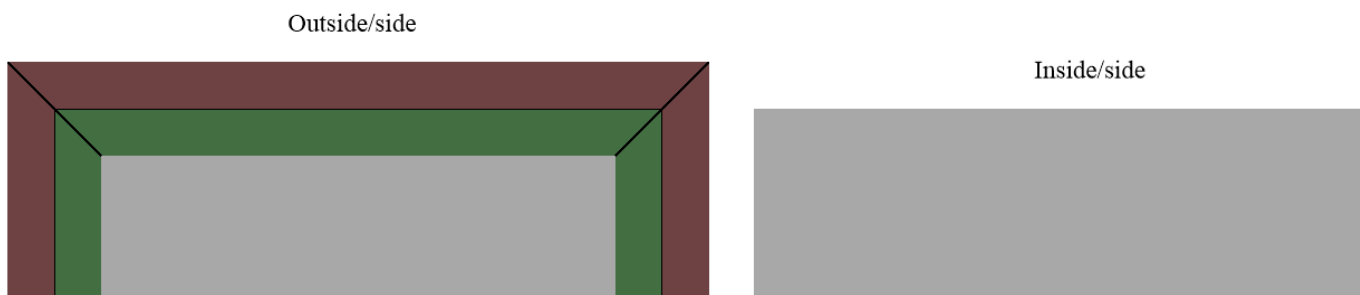
I now have models of the outer and inner parts. It is time to put them together.

The Design Strategy

Looking at the end of the box, I see the outside/end around the Inside/end.



The brown border is the material thickness. The green is the remaining face of the outside/end.

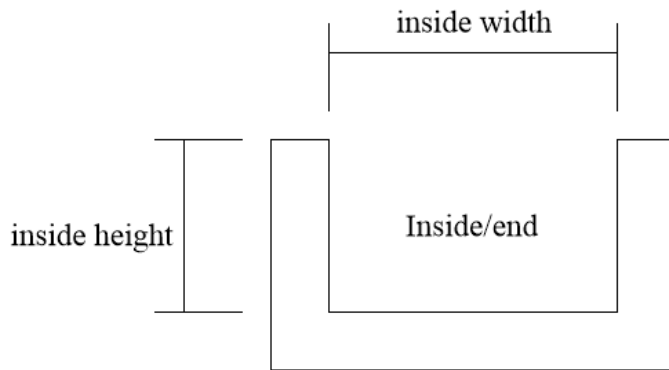


Looking at the side of the box, I see the Outside/side closely fitting over the Inside/side. The inside is as large as possible, and the outside is as small as possible.

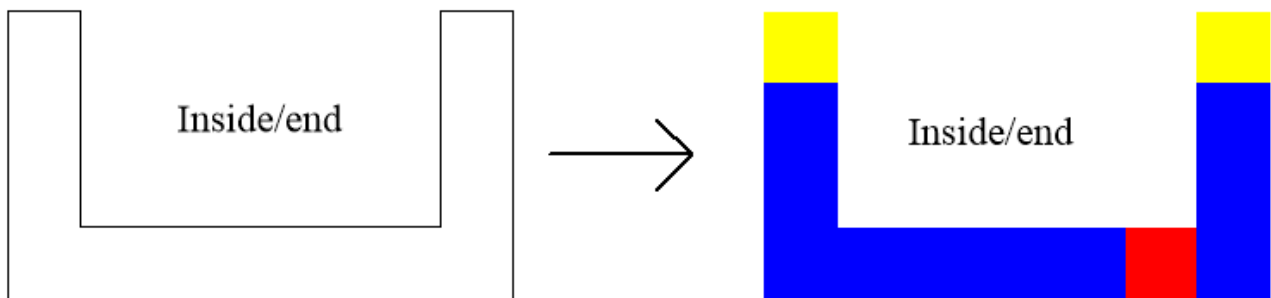
The inside part just fits inside the outside part. Any gap between them is due to cutting and bending variation encountered while making the outside/end

These insights will guide my choice of errors when I do the analysis.

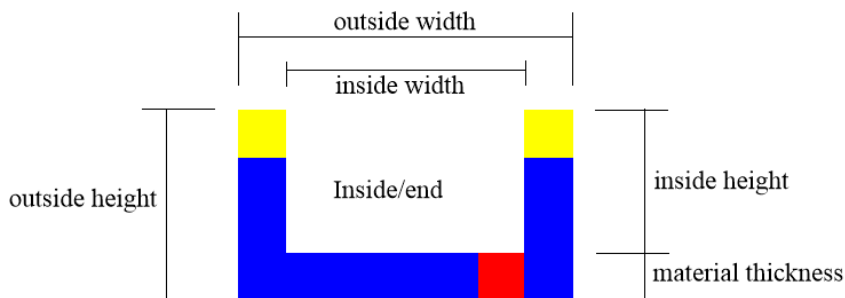
High-Level Design of the Inside Part



My specifications include the inside height and inside width. I must not bend up a box that is smaller than these dimensions.

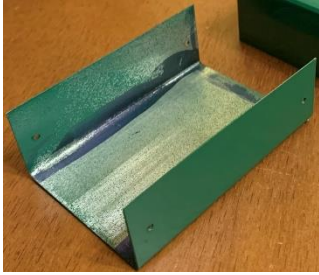


Recall that we have yellow and red error tolerances.

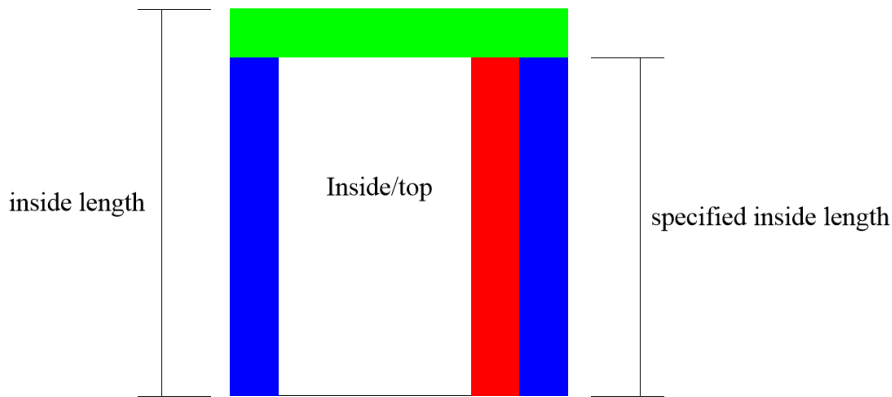
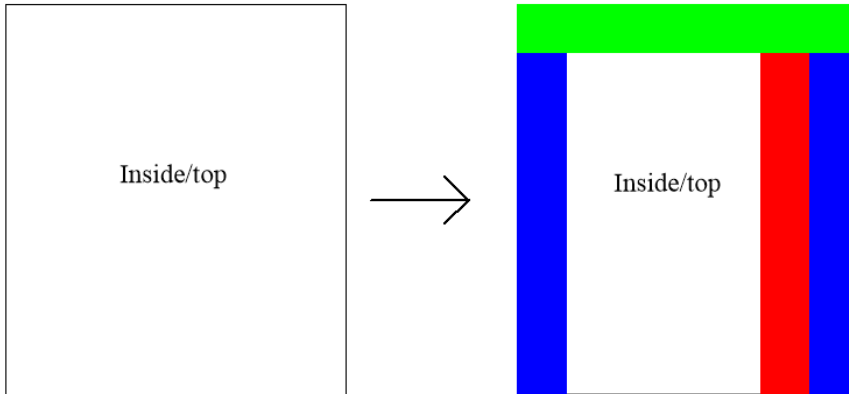


The maximum outside height of this part equals the inside height plus material thickness plus the maximum yellow error contribution.

The maximum outside width equals the inside width plus twice the material thickness plus the maximum red error contribution.



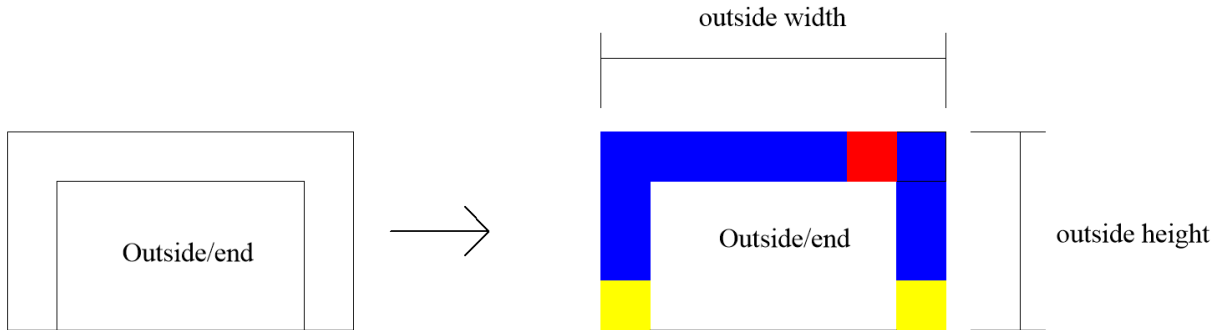
Looking down on the inside part, I see the inside length.



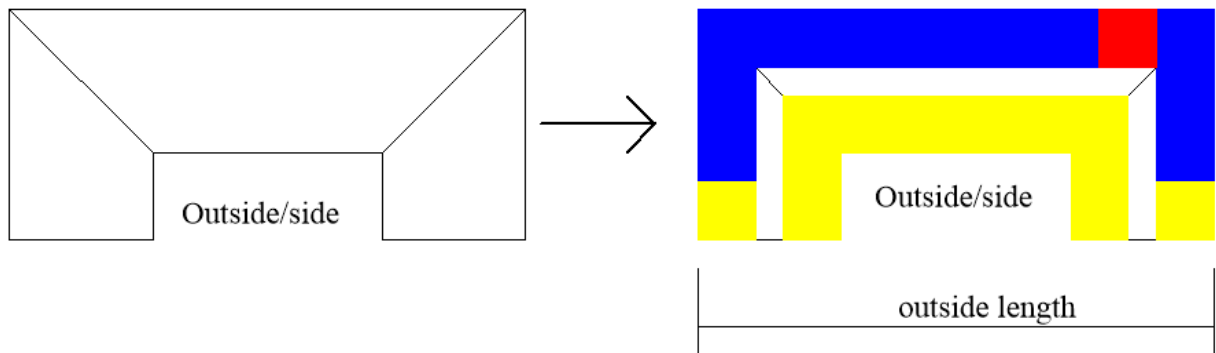
The maximum inside length equals the specified inside length plus the maximum green error contribution.

High-Level Design of the Outside Part

Looking at the end of the outside part, I see a solid face. To see the detail, I am looking at an X-ray view.



The minimum outside width equals the measured inside width of the inside part plus twice the material thickness. The red error contribution is zero.



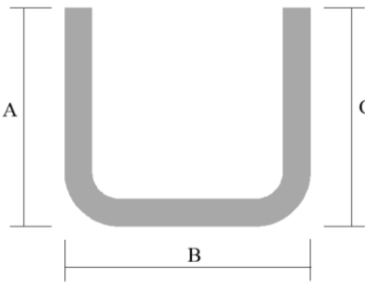
The minimum outside length equals the measured inside width plus twice the material thickness. The red error contribution is zero.

The Key Equations

The concepts employed here are from

<https://rick.sparber.org/SheetMetalBending.pdf>.

If the following confuses you, I suggest you go back and read this article.



Before I can bend sheet metal, I must measure my brake and see how it bends the selected material:

$$k = \frac{(A + B + C) - OAL}{2} \quad (1)$$

OAL is the measured length of the sample before bending.

k is a constant that is a function of the brake and the material's thickness. After sufficient testing, I will find

$$k_{expected} = k_{mean} \pm e_k \quad (2)$$

e_k is the variability of a given brake to make a bend in a given material. This equation is saying that my expected value for k is bounded by $(k_{mean} - e_k)$ and $(k_{mean} + e_k)$.

A rearranging of

$$k = \frac{(A + B + C) - OAL}{2} \quad (1)$$

is the central equation relating the desired dimensions of the finished part to the length of the material before bending a “U” shape with equal sides ($A = C$):

$$OAL = 2A + B - 2k \quad (3)$$

A and B are specified, so are exact. k is ideally a constant. It lets me calculate the needed OAL.

An equation that defines the sight line distance will be presented later.

Variational Analysis

I am making this box from two pieces. No matter how the errors mix together, these pieces have to fit together. This means that the inside piece must have inside dimensions that are

- at least as tall as the specified height,
- as wide as the specified width, and
- as long as the specified length.

Due to these errors, I must allow the actual height, width, and length to be larger than specified.

The design of the outside part follows the same logic as used for the inside part. This means that the outside piece must have inside dimensions that are

- at least as tall as the measured height of the inside part,
- as wide as the measured width of the inside part, and
- as long as the measured length of the inside part.

I found it very easy to get confused as I stumbled through all of these errors. Periodically referring back to a picture of the finished box was helpful.

Characterizing The Brake

I characterize the brake by bending the selected material and then calculating k . See page 3 for the procedure.

For each sample, I plug into

$$k = \frac{(A + B + C) - OAL}{2} \quad (1)$$

the measured values of A, B, C, and OAL, which should be accurate to within ± 0.001 – inches.

After sufficient testing, the spreadsheet estimates how k behaves. Empirically, I found that five samples gave accurate results.

$$k_{\text{expected}} = k_{\text{mean}} \pm e_k \quad (4)$$

$$k_{\text{expected}} = k_{\text{mean}} \pm e_k \quad (2)$$

where

$$k_{\text{mean}} = \frac{k_{\text{maximum}} + k_{\text{minimum}}}{2} \quad (5)$$

$$e_k = k_{\text{mean}} - k_{\text{minimum}} \quad (6)$$

Note that my definition of “mean” is not standard.

e_k is the variability of a given brake to make a bend in a given material. (4) is saying that my expected value for k is bounded by $(k_{\text{mean}} - e_k)$ and $(k_{\text{mean}} + e_k)$.

Characterizing My Skill

The standard equation for sight line distance is

$$SLD_{needed} = A_{needed} - MT \quad (7)$$

On my homemade brake, I have found that the fingers bend a little, and my ability to position the sight line is not perfect³. Therefore, the standard equation does not work well for me. Instead, I use

$$SLD_{needed} = A_{needed} - m \quad (8)$$

which can be rearranged to tell me m by measuring A and knowing the needed SLD.

$$m = A_{measured} - SLD_{needed} \quad (9)$$

$$SLD_{needed} = A_{needed} - m \quad (8)$$

equation also applies to dimension C:

$$m = C_{measured} - SLD_{needed} \quad (10)$$

With five test pieces, I get ten values for m . I again calculate the mean and the variation

$$m_{expected} = m_{mean} \pm e_m \quad (11)$$

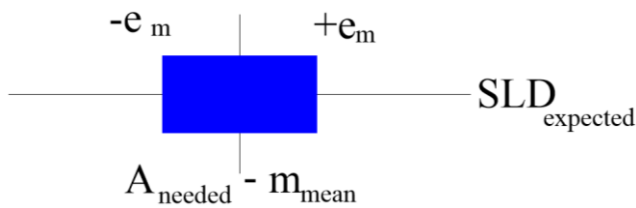
Knowing the range of values for m lets me predict the range of values for the SLD.

$$SLD_{expected} = A_{needed} - m_{expected} \quad (12)$$

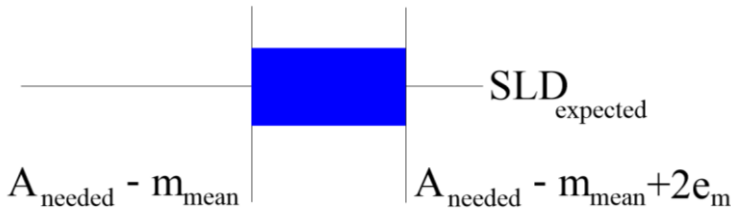
or

$$SLD_{expected} = A_{needed} - \{m_{mean} \pm e_m\} \quad (13)$$

³ For example, when bending stock 0.020-inches thick ($MT = 0.020$), I found m was 0.0385. This difference is repeatable, which is all that matters when shooting for predictable results.



This is telling me that $SLD_{expected}$ is centered at $A_{needed} - m_{mean}$ and can vary from this point by plus or minus e_m .



I am stuck with this variation in m but can add an offset to A_{needed} to ensure it is always equal to or greater than $A_{needed} - m_{mean}$.

All of my variation has been pushed to the high side. Where before, the maximum value of the expected SLD was

$$A_{needed} - \{m_{mean} + e_m\},$$

it is now

$$A_{needed} - \{m_{mean} + 2e_m\}.$$

This is useful when I must meet a requirement that says a given dimension must be at least the specified value.

A second challenge to my skill is how accurately I can cut the sheet metal. I know OAL_{needed} and also know $OAL_{measured}$. Each test strip will provide one sample of OAL_{error} :

$$OAL_{error} = OAL_{needed} - OAL_{measured} \quad (14)$$

Given enough test strips, I can build up a profile of my error:

$$OAL_{expected\ error} = OAL_{mean\ error} \pm e_{OAL} \quad (15)$$

This lets me predict how my measured OAL will vary:

$$OAL_{expected} = OAL_{needed} + OAL_{expected\ error} \quad (16)$$

or

$$OAL_{expected} = OAL_{needed} + \{OAL_{mean\ error} \pm e_{OAL}\} \quad (17)$$

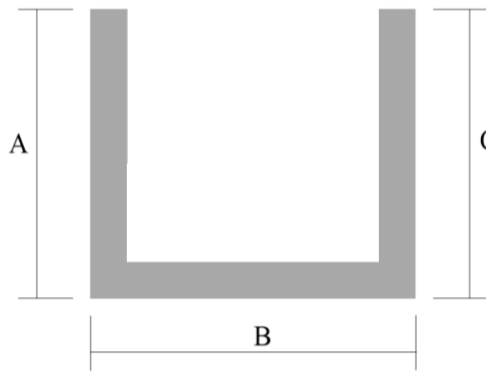
As with the SLD, I can offset OAL to ensure it is never less than a given value.

With expected values for k , m , and $OAL_{expected\ error}$, I'm ready to look at bending up the box, so the inside and outside parts always fit.

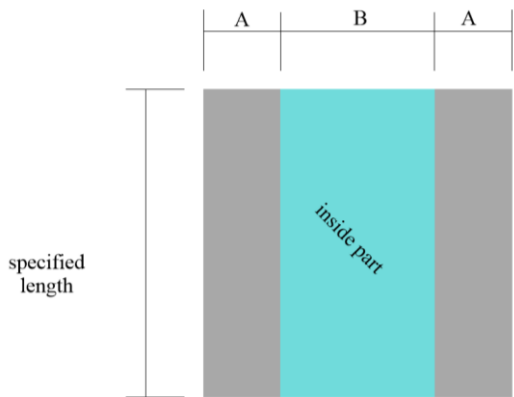
The Inside Part



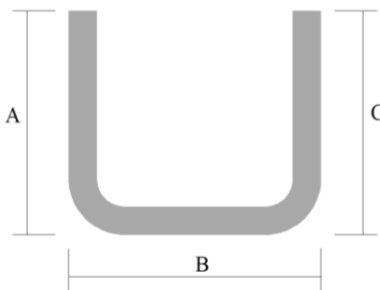
Looking at the inside part from the end, I see a U shape.



Just to review: ideally, the corners are square. The outside height is A, which is also C. The outside width is B.



This U is bent up from sheet metal.



In reality, the corners cannot be square because the metal would crack. Instead, I have a radius that has the effect of changing the layout on the sheet metal. Here is where OAL and SLD come into the picture.

Sight Line Distance for A

$$SLD_{needed} = A_{needed} - m \quad (8)$$

This is telling me that I can calculate SLD_{needed} because I know $A_{specified}$ and “m”. Well, $A_{specified}$ has no error, but m is derived from my test bends:

$$m_{expected} = m_{mean} \pm em \quad (11)$$

I need to settle on a value for m in order to scribe my sight line. Why not use the mean? Then the SLD will also be the mean.

$$SLD_{mean} = A_{specified} - m_{mean} \quad (18)$$

I scribe my sight line and make the bend. What should I expect for the resulting value for A?

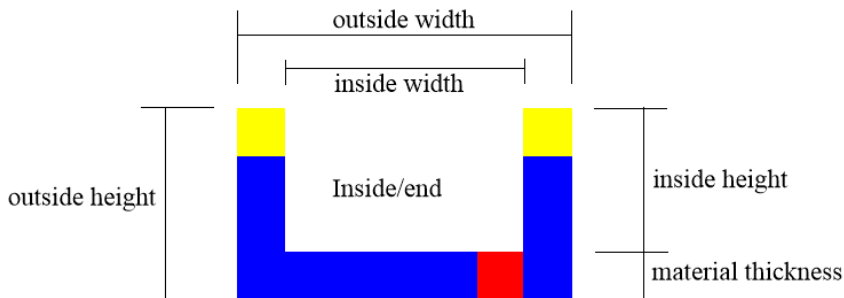
$$A_{expected} = SLD_{mean} + m \quad (19)$$

$$A_{expected} = SLD_{mean} + \{m_{mean} \pm e_m\} \quad (20)$$

$$A_{expected} = (SLD_{mean} + m_{mean}) \pm e_m \quad (21)$$

$$A_{expected} = (A_{specified}) \pm e_m \quad (22)$$

The variation in m becomes the variation in the expected value of A.



Recall from page 13 that the outside height equals the inside height plus MT. The inside height is a requirement: I must have at least this much height:

$$A \geq \text{inside height} + MT \quad (23)$$

This gives me permission to add an offset to A. I know, from (), that the expected variation in A is $\pm e_m$. If I set $A = \textit{inside height} + MT$, my $A_{\textit{expected}}$ could be as small as $A - e_m$. But then, my $A_{\textit{expected}}$ would be less than $\textit{inside height} + MT$ so would not meet the specification. Instead, I choose to add e_m to my specified value for A:

$$A_{\textit{needed}} = A_{\textit{specified}} + e_m \quad (24)$$

$$A_{\textit{needed}} = \textit{inside height} + MT + e_m \quad (25)$$

$A_{\textit{needed}}$ will now replace $A_{\textit{specified}}$ in ():

$$A_{\textit{expected}} = (\textit{inside height} + MT + e_m) \pm e_m \quad (26)$$

Which reduces to

$$A_{\textit{expected}} = \textit{inside height} + MT + 2e_m / -0 \quad (27)$$

We know that

$$SLD_{\textit{mean}} = A_{\textit{specified}} - m_{\textit{mean}} \quad (18)$$

$A_{\textit{specified}}$ is now $A_{\textit{needed}}$:

$$A_{\textit{needed}} = \textit{inside height} + MT + e_m \quad (25)$$

So I can specify my needed SLD:

$$SLD_{\textit{needed}} = \textit{inside height} + MT + e_m - m_{\textit{mean}} \quad (28)$$

$$SLD_{\textit{mean}} = A_{\textit{specified}} - m_{\textit{mean}} \quad (18)$$

To recap, I scribe my sight line using $m_{\textit{mean}}$ and can then expect A to be between ($\textit{inside height} + MT$) and ($\textit{inside height} + MT + 2e_m$).

Over All Length

$$OAL = 2A + B - 2k \quad (3)$$

The first step is to fill in what I know.

$$A_{expected} = \text{inside height} + MT + 2e_m / -0 \quad (27)$$

$$A_{expected} = \text{inside height} + MT + 2e_m / -0 \quad (27)$$

$$k_{expected} = k_{mean} \pm e_k \quad (2)$$

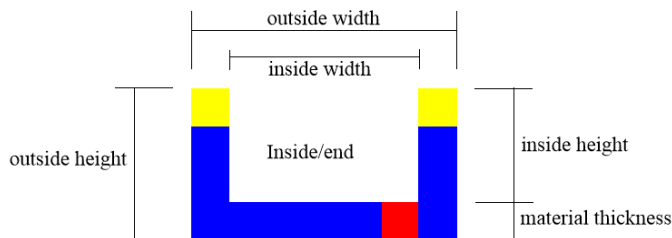
$$k_{expected} = k_{mean} \pm e_k \quad (4)$$

Which gives me

$$OAL = 2\{\text{inside height} + MT + 2e_m / -0\} + B - 2\{k_{mean} \pm e_k\} \quad (29)$$

Solving for B, I get

$$B = OAL - 2\{\text{inside height} + MT + 2e_m / -0\} + 2\{k_{mean} \pm e_k\} \quad (30)$$



Recall that the outside width equals the inside width plus $2MT$. To not violate the inside width specification, I limit B:

$$B \geq \text{inside width} + 2MT \quad (31)$$

Equating (30) and (31) gives me

$$OAL - 2\{\text{inside height} + MT + 2e_m / -0\} + 2\{k_{mean} \pm e_k\} \geq \text{inside width} + 2MT \quad (32)$$

Solving for OAL, I get

$$\begin{aligned} OAL \geq & 2 \{ \textit{inside height} + MT + 2e_m / -0 \} \\ & - 2 \{ k_{\text{mean}} \pm e_k \} + \textit{inside width} \\ & + 2MT \end{aligned} \quad (33)$$

The right side of this equation tells me what causes OAL to change. The inequality says that OAL must be equal to or greater than the right side. Under all situations, I want OAL to be large enough to meet the specifications, so I choose my various error limits to give me the largest number.

$$\begin{aligned} OAL \geq & 2 \{ \textit{inside height} + MT + 2e_m \} \\ & - 2 \{ k_{\text{mean}} - e_k \} + \textit{inside width} \\ & + 2MT \end{aligned} \quad (34)$$

From assessing my own skill at cutting to a needed OAL, I know

$$\begin{aligned} OAL_{\textit{expected}} &= OAL_{\textit{needed}} \\ &+ \{ OAL_{\textit{mean error}} \pm e_{OAL} \} \end{aligned} \quad (17)$$

This is telling me that the OAL I can expect to get will equal my needed OAL offset by $OAL_{\textit{mean error}}$. Additionally, it can vary by $\pm e_{OAL}$.

If I subtract $OAL_{\textit{mean error}}$ from OAL, (17) will add it back in, so I end up with what I really wanted. If I also added in e_{OAL} to OAL, this variation in (17) can make $OAL_{\textit{expected}}$ be larger than needed but never smaller. I thereby arrive at my $OAL_{\textit{needed}}$:

$$\begin{aligned} OAL_{\textit{needed}} &= 2 \{ \textit{inside height} + MT + 2e_m \} \\ &- 2 \{ k_{\text{mean}} - e_k \} + \textit{inside width} \\ &+ 2MT - OAL_{\textit{mean error}} + e_{OAL} \end{aligned} \quad (35)$$

Which simplifies to

$$\begin{aligned}
 OAL_{needed\ width} & & (36) \\
 &= 2(\textit{inside height}) + (\textit{inside width}) \\
 &+ 4MT + 4e_m - 2k_{mean} + 2e_k \\
 &- OAL_{mean\ error} + e_{OAL}
 \end{aligned}$$

I have also added “width” to OAL_{needed} because I will later be dealing with the inside part’s $OAL_{needed\ length}$.

Do I get an acceptable range of values for OAL? Plug (36) into (17):

$$\begin{aligned}
 OAL_{expected} &= \{2(\textit{inside height}) + (\textit{inside width}) & (37) \\
 &+ 4MT + 4e_m - 2k_{mean} + 2e_k \\
 &- OAL_{mean\ error} + e_{OAL}\} \\
 &+ OAL_{mean\ error} \pm e_{OAL}
 \end{aligned}$$

Which simplifies to

$$\begin{aligned}
 OAL_{expected} &= 2(\textit{inside height}) + (\textit{inside width}) & (38) \\
 &+ 4MT - 2k_{mean} \\
 &+ 4e_m + 2e_k + 2e_{OAL}/_{-0}
 \end{aligned}$$

Notice that $OAL_{mean\ error}$ is gone and variation in $2e_{OAL}$ can only make $OAL_{expected}$ larger.

Circling back to

$$\begin{aligned}
 B &= OAL - 2\{\textit{inside height} + MT + 2e_m/_{-0}\} & (30) \\
 &+ 2\{k_{mean} \pm e_k\}
 \end{aligned}$$

I can plug $OAL_{expected}$ in for OAL and get the expected range of values for B:

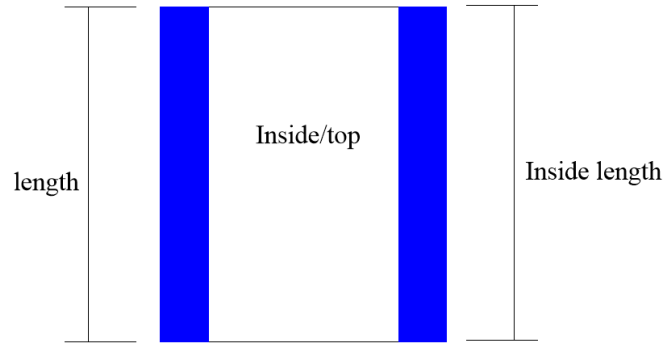
$$\begin{aligned}
 B_{expected} &= \{2(\textit{inside height}) + (\textit{inside width}) & (39) \\
 &+ 4MT + 4e_m - 2k_{mean} \\
 &+ 2e_k + 2e_{OAL}/_{-0}\} - 2\{\textit{inside height} \\
 &+ MT + 2e_m/_{-0}\} + 2\{k_{mean} \pm e_k\}
 \end{aligned}$$

Which simplifies to

$$B_{expected} = \textit{inside width} + 2MT + 2e_{OAL}/_{-0} + 4e_m/_{-0} + 4e_k/_{-0} \quad (40)$$

From (31) I know that $B_{expected}$ must not be smaller than $\textit{inside width} + 2MT$. As OAL, m , and k vary, $B_{expected}$ can only get larger. So, at least this looks reasonable.

Inside Length



When I cut an OAL, I know there is variation:

$$OAL_{expected} = OAL_{needed} + \{OAL_{mean\ error} \pm e_{OAL}\} \quad (17)$$

This equation tells me that I may need OAL_{needed} but I will get $OAL_{expected}$. The smallest $OAL_{expected}$ is when I am at the negative value for e_{OAL}

$$OAL_{minimum\ expected} = OAL_{needed} + \{OAL_{mean\ error} - e_{OAL}\} \quad (41)$$

To meet the inside length specification, I set $OAL_{minimum\ expected} = \textit{inside length}$. I can then have

$$\textit{inside length} = OAL_{needed} + \{OAL_{mean\ error} - e_{OAL}\} \quad (42)$$

Now I can solve for OAL_{needed} and be confident that the OAL will always be equal to or greater than the specified inside length. I renamed it too.

$$\begin{aligned} OAL_{needed\ length} & & (43) \\ &= \textit{inside length} \\ &- OAL_{mean\ error} + e_{OAL} \end{aligned}$$

Now that I have $OAL_{needed\ length}$, I can find out how it varies.

$$\begin{aligned} OAL_{expected} &= OAL_{needed} & (17) \\ &+ \{OAL_{mean\ error} \pm e_{OAL}\} \end{aligned}$$

Set OAL_{needed} equal to $OAL_{needed\ length}$:

$$\begin{aligned} OAL_{expected\ length} & & (44) \\ &= \{\textit{inside length} \\ &- OAL_{mean\ error} + e_{OAL}\} \\ &+ \{OAL_{mean\ error} \pm e_{OAL}\} \end{aligned}$$

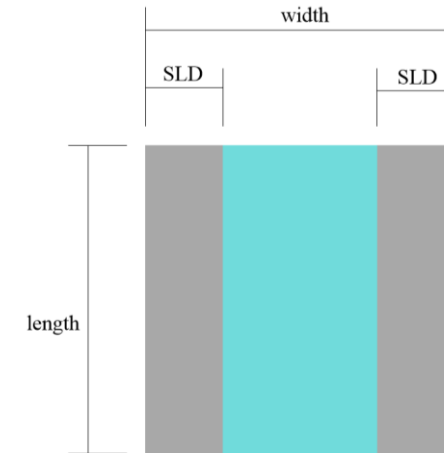
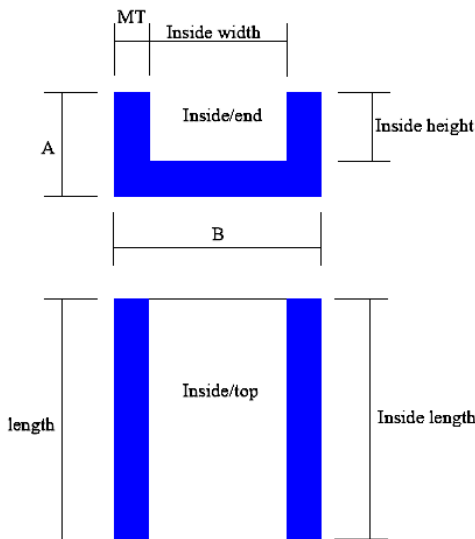
Which simplifies to

$$OAL_{expected\ length} = \textit{inside length} + 2e_{OAL}/0 \quad (45)$$

$$\begin{aligned} OAL_{needed\ length} & & (43) \\ &= \textit{inside length} \\ &- OAL_{mean\ error} + e_{OAL} \end{aligned}$$

Summary: Design Equations for the Inside Part

You specify the inside width, height, and length plus material thickness.



These equations tell you to cut a piece of sheet metal that is $OAL_{needed\ width}$ by $OAL_{needed\ length}$.

Then you must scribe two sight lines that are SLD_{needed} distance from the edges.

$$OAL_{needed\ width} = 2(\text{inside height}) + (\text{inside width}) + 4MT + 4e_m - 2k_{mean} + 2e_k - OAL_{mean\ error} + e_{OAL} \quad (36)$$

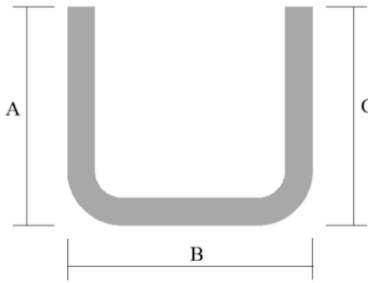
$$OAL_{needed\ length} = \text{inside length} - OAL_{mean\ error} + e_{OAL} \quad (43)$$

$$A_{needed} = \text{inside height} + MT + e_m \quad (25)$$

$$SLD_{needed} = \text{inside height} + MT + e_m - m_{mean} \quad (28)$$

$$A_{expected} = \text{inside height} + MT + 2e_m / -0 \quad (27)$$

With the narrow segment under the finger and the sight line aligned with the end of the finger, make the two bends.



Then measure the part's A, B, C, and length. A is defined as being taller than C.

$$B_{expected} = \textit{inside width} + 2MT + 2e_{OAL}/_{-0} + 4e_m/_{-0} + 4e_k/_{-0} \quad (40)$$

Bench Testing What I Have So Far

First, I had to cut five test strips to get the various derived parameters. They are of the same material that I will later use for the part. I marked each one to have an OAL of 3.000-inches. After cutting, I measured each OAL and entered them into the spreadsheet. Then I scribed my SLD, which was specified to be 1.000-inches from each end.

| Box Bending | | | | | | | | | |
|--|--------------|------------|-------------------|------------|------------------------------------|---------------------|---------------------|-----------|--|
| [1] Characterize your brake with the selected sheetmetal | | | | | use OAL = 3.000 use SLD = 1.000 | | | | |
| sample | measured OAL | measured A | measured B | measured C | calculated k | calculated m from A | calculated m from C | OAL error | |
| 1 | 3.005 | 1.018 | 1.086 | 1.035 | 0.066 | 0.018 | 0.035 | -0.005 | |
| 2 | 2.992 | 1.052 | 1.035 | 1.033 | 0.064 | 0.052 | 0.033 | 0.009 | |
| 3 | 2.980 | 1.037 | 1.024 | 1.041 | 0.061 | 0.037 | 0.041 | 0.021 | |
| 4 | 3.003 | 1.026 | 1.046 | 1.028 | 0.049 | 0.026 | 0.028 | -0.002 | |
| 5 | 2.986 | 1.032 | 1.021 | 1.030 | 0.049 | 0.032 | 0.030 | 0.015 | |
| mean k | 0.057 | | OAL mean error | 0.008 | | | | | |
| +/-k variation | 0.009 | | +/- OAL variation | 0.013 | | | | | |
| mean m | 0.035 | | | | | | | | |
| +/-m variation | 0.017 | | | | | | | | |

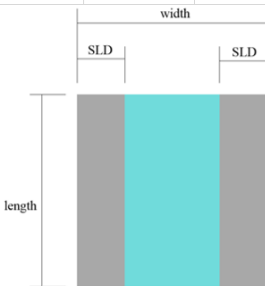
[2] Characterize your selected material

| | |
|----------------------|-------|
| material thickness = | 0.020 |
|----------------------|-------|

[3] Define needed internal dimensions of the box

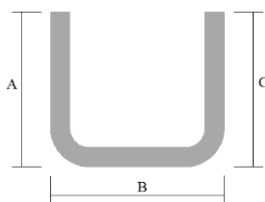
| | |
|----------|-------|
| height = | 1.000 |
| width = | 1.000 |
| length = | 1.000 |

| | | | |
|--------------------|-------|----------------|-------|
| A expected max | 1.055 | B expected max | 1.170 |
| A expected min | 1.020 | B expected min | 1.040 |
| [4] inside part: | | | |
| SLD = | 1.003 | | |
| width = | 3.057 | | |
| length = | 1.005 | | |
| expected width max | 3.083 | | |
| expected width min | 3.057 | | |



My material is 0.020-inches thick. I wanted to bend a U with an inside height and width of 1.000-inches. The spreadsheet told me to cut the width to 3.057-inches. The spreadsheet predicted it would be between 3.057 and 3.083-inches. After cutting, I measured it at 3.0590-inches.

The sight lines are to be 1.003-inches from the ends. I didn't test the length.



After bending, I measured A. B. and C.

The spreadsheet predicted that A and C should be between 1.020 and 1.055. I measured 1.0245 for A and 1.0375 for C.

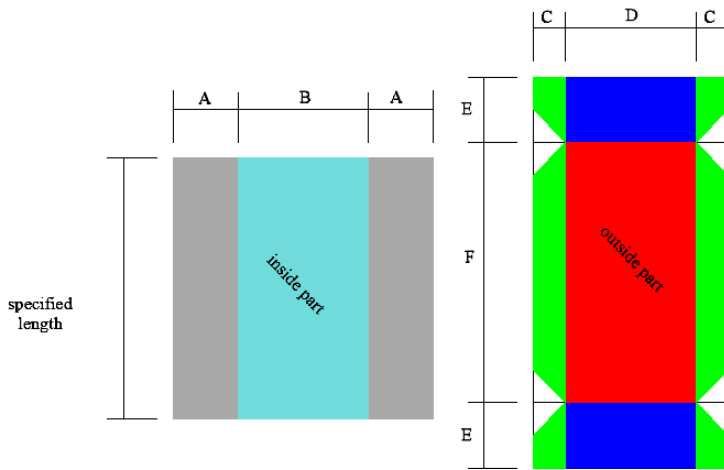
B should be between 1.040 and 1.170. I measured 1.087.

This does not stand as a proof but is a good sanity test of the equations.

Nomenclature



When I was just talking about the inside part, it was sufficient to just talk about A, B, and OAL. The outside part is twice as complicated.

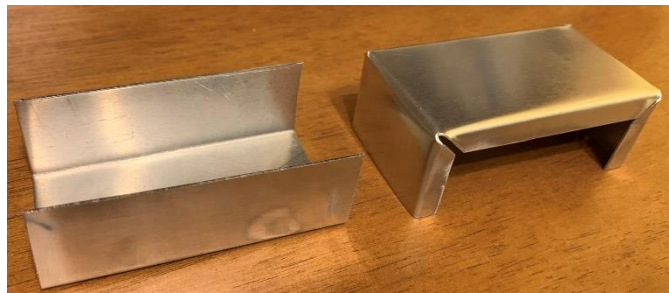


I will be folding the sheet metal around the ends of the inside part and also around the sides. More letters have been added to keep this all straight.

Thankfully, the equations developed for A and B apply to C and D and also to E and F.

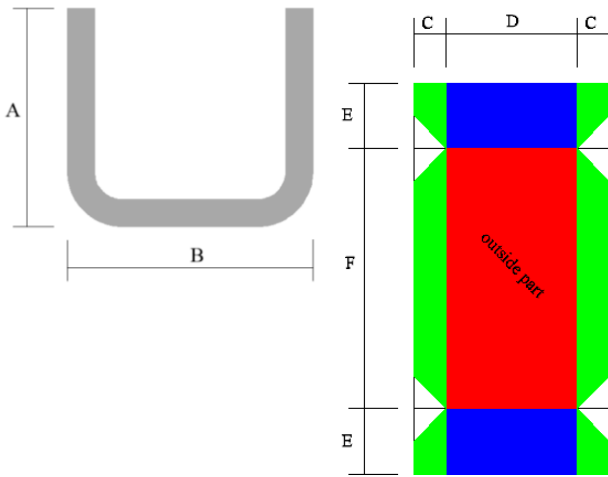
As a final step, I will add notches, so the corners don't bind up. These will not affect the above dimensions.

The General Layout of the Outside Part



The gray "U" is an end view of the inside part. The green and blue box above and to the right of it is an end view of the top part. The blue is the face of the end, and the green is an end view of the material thickness, which wraps around the sides.

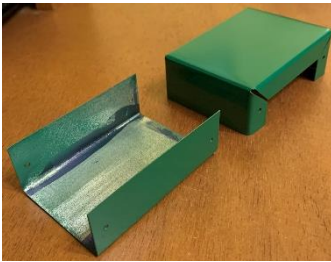
Needed E SLD



When the two E segments are bent, they must cover the ends of the inside part. This means that the minimum value for E must equal the measured value of A plus MT because E is the outside dimension.

I can, therefore say that

$$E_{\text{minimum needed}} = A + MT \quad (46)$$



Referring back to the forming of A:

$$A_{\text{expected}} = \text{inside height} + MT + 2e_m / -0 \quad (27)$$

I can see that

$$A_{\text{minimum expected}} = \text{inside height} + MT \quad (47)$$

The inside part's "inside height" is replaced by A. $A_{\text{minimum expected}}$ is replaced by $E_{\text{minimum expected}}$. It follows that I can say

$$E_{\text{expected}} = A + MT + 2e_m / -0 \quad (48)$$

This tells me that E_{expected} can be any value between $(A + MT)$ and $(A + MT + 2e_m)$. I want to set my SLD to the middle of this range:

$$E_{\text{specified}} = A + MT + e_m \quad (49)$$

Now, I can define my needed SLD. Looking back at A, I see

$$SLD_{\text{mean}} = A_{\text{specified}} - m_{\text{mean}} \quad (18)$$

Which becomes

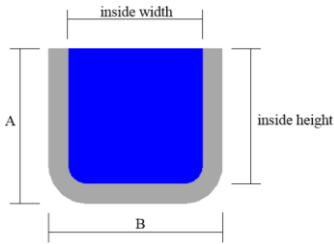
$$SLD_{E \text{ mean}} = E_{\text{specified}} - m_{\text{mean}} \quad (50)$$

I put (49) into (50) and rename “mean” to “needed”:

$$SLD_{E \text{ needed}} = (A + MT + e_m) - m_{\text{mean}} \quad (51)$$

OAL_{EFE}

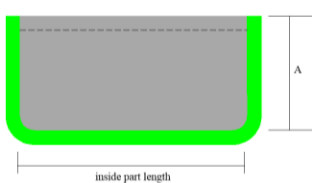
I went back to the inside part to repurpose its equation.



Notice that the inside part (gray) wraps around the inside height and inside width (blue).

$$\begin{aligned}
 OAL_{needed} = & 2\{inside\ height + MT + 2e_m\} \\
 & - 2\{k_{mean} - e_k\} + inside\ width \\
 & + 2MT - OAL_{mean\ error} + e_{OAL}
 \end{aligned} \tag{35}$$

Now consider how the outside part wraps around the inside part:



This is an up-side-down, X-ray, side view of the outside part (green) fitted around the inside part (gray).

The inside height, shown above, is in the same relative location as the A dimension.

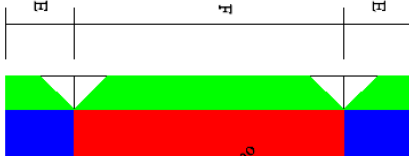


The inside width is in the same relative location as the inside part length. This tells me what I must substitute in $OAL_{ABA\ needed}$ to get $OAL_{EFE\ needed}$.

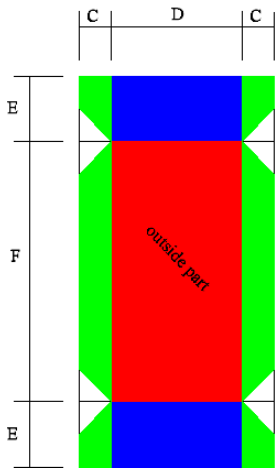
$$\begin{aligned}
 OAL_{needed\ width} = & 2(inside\ height) + (inside\ width) + 4MT + \\
 & 4e_m - 2k_{mean} + 2e_k - OAL_{mean\ error} + e_{OAL}
 \end{aligned} \tag{36}$$

Using the above-mentioned substitutions, I get

$$\begin{aligned}
 OAL_{EFE} \text{ needed width} &= 2(A) + (\text{inside part length}) + 4MT \\
 &+ 4e_m - 2k_{\text{mean}} + 2e_k \\
 &- OAL_{\text{mean error}} + e_{OAL}
 \end{aligned}
 \tag{52}$$



Needed C SLD



The tab that runs around the flanks of the outside part captures the inside part. It also provides an area for the sheet metal screws. C is the width of each tab and is user-specified.

Referring back to A, I have

$$SLD_{\text{mean}} = A_{\text{specified}} - m_{\text{mean}}
 \tag{18}$$

Which becomes

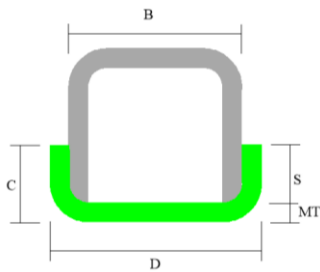
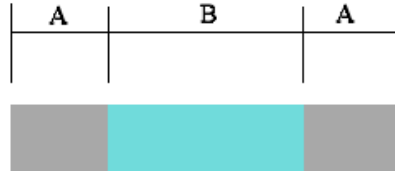
$$SLD_{C \text{ needed}} = C - m_{\text{mean}}
 \tag{53}$$

C is not a critical dimension, so I didn't bother to characterize how it varies.

OAL_{CDC}

Going back to the inside part, I have

$$\begin{aligned}
 OAL_{needed} = & 2\{inside\ height + MT + 2e_m\} \\
 & - 2\{k_{mean} - e_k\} + inside\ width \\
 & + 2MT - OAL_{mean\ error} + e_{OAL}
 \end{aligned} \tag{35}$$



This is an up-side-down, X-ray, end view of the outside part (green) fitted around the inside part (gray).



The inside height, shown above, is in the same relative location as the S dimension, and

$$\begin{aligned}
 S + MT &= C \\
 \text{or} \\
 S &= C - MT
 \end{aligned}$$

The inside width is in the same relative location as the B dimension.

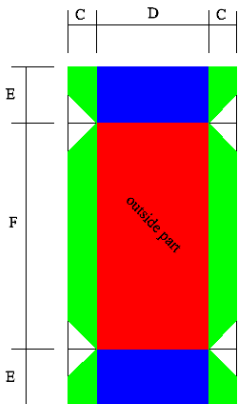
This tells me what I must substitute in $OAL_{ABA\ needed}$ to get $OAL_{CDC\ needed}$

$$\begin{aligned}
 OAL_{CDC\ needed\ width} & \\
 = & 2(C - MT) + (B) + 4MT + 4e_m \\
 & - 2k_{mean} + 2e_k - OAL_{mean\ error} + e_{OAL}
 \end{aligned} \tag{54}$$

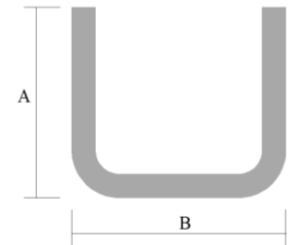
which simplifies to

$$\begin{aligned}
 OAL_{CDC\ needed\ width} & \\
 = & 2C + (B) + 2MT + 4e_m - 2k_{mean} \\
 & + 2e_k - OAL_{mean\ error} + e_{OAL}
 \end{aligned} \tag{55}$$

Summary: Design Equations For The Outside Part



A and B are measured on the inside part. You choose C.



$$\begin{aligned} \text{OAL}_{\text{CDC needed width}} &= 2C + (B) + 2MT + 4e_m - 2k_{\text{mean}} \\ &+ 2e_k - \text{OAL}_{\text{mean error}} + e_{\text{OAL}} \end{aligned} \quad (55)$$

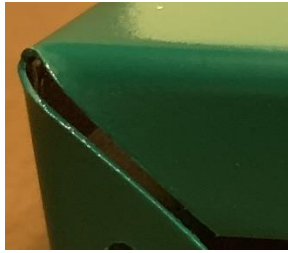
$$\begin{aligned} \text{OAL}_{\text{EFE needed width}} &= 2(A) + (\text{inside part length}) + 4MT \\ &+ 4e_m - 2k_{\text{mean}} + 2e_k \\ &- \text{OAL}_{\text{mean error}} + e_{\text{OAL}} \end{aligned} \quad (52)$$

$$\text{SLD}_{\text{C needed}} = C - m_{\text{mean}} \quad (53)$$

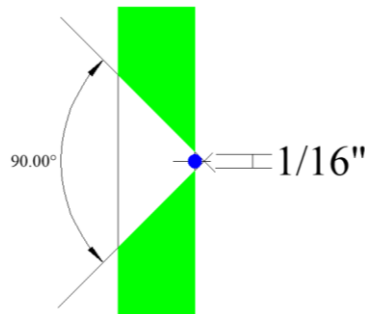
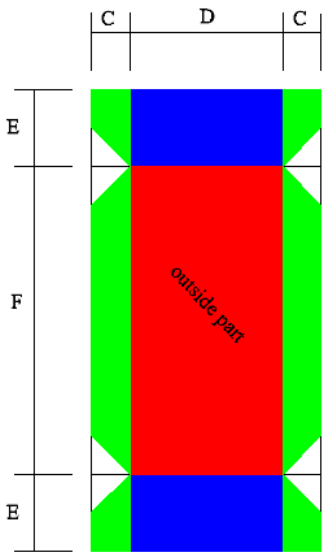
$$\text{SLD}_{\text{E mean}} = E_{\text{specified}} - m_{\text{mean}} \quad (50)$$

$$\text{SLD}_{\text{E needed}} = (A + MT + e_m) - m_{\text{mean}} \quad (51)$$

Notching and Drilling



To fold up the outer part, I must leave room for C at the corners. If I just notch out 90°, the edges will hit as I bend slightly past 90° and then let go to get 90°. How much “over bend” you need will depend on the material.



A reasonable starting point is to offset the 90° lines. One way to do this is to drill a hole, and then scribe lines 90° apart tangent to it.



You may also want to add holes for sheet metal screws. I suggest drilling tap holes in the outside part before bending. Then bend up the inside and outside parts. After assembly, drill through these holes into the inside part. With the inside holes drilled, disassemble the box. Then, open out the holes in the outside part to clearance.

A Test Project Box



I chose to bend up a box with inside dimensions of 1-inch by 2-inches by 3-inches. This enables me to drop in a precision ground 1-2-3 block to verify the fit.

| | |
|-----|--|
| [3] | Define needed internal dimensions of the box |
| | height = 1.000 |
| | width = 2.000 |
| | length = 3.000 |

After running my five test bends and recording the measurements, I input my material thickness and inside dimensions.

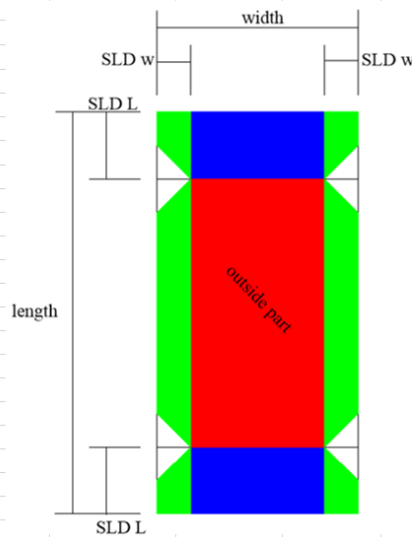
| | | |
|-----|--------------|-------|
| [4] | inside part: | |
| | SLD = | 1.003 |
| | width = | 4.057 |
| | length = | 3.005 |

Out came the width and length of the blank along with the SLD to be scribed on each end.

| | | |
|-----|-------------------|-------|
| [5] | chosen C = | 0.500 |
| | measured A = | 1.035 |
| | measured B = | 2.085 |
| | measured length = | 3.006 |

I bent up the inside part and recorded the dimensions. I also chose the tab, C, to be 0.500-inches wide.

| | | |
|-----|---------------|-------|
| [6] | outside part: | |
| | length = | 5.132 |
| | SLD L = | 1.064 |
| | width = | 3.172 |
| | SLD w = | 0.492 |



Out popped the dimensions of the outside part.

The resulting inside and outside parts fit together with a nice, sliding fit. My 1-2-3 block fits inside with a little margin.

I welcome your comments and questions.

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